

“Ligand” Binding

“The secret of life is molecular recognition; the ability of one molecule to “recognize” another through weak bonding interactions.”

Linus Pauling at the 25th anniversary of the Institute of Molecular Biology at the University of Oregon

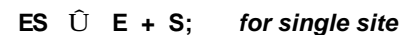
Binding is the first step necessary for a biological response. Before macromolecules can perform a function, they normally must interact with a specific ligand. In some cases like myoglobin, binding and subsequent release of the ligand might be the sole function of the macromolecule. To understand binding, we must consider the equilibria involved, how binding is affected by ligand and macromolecule concentration, and how to experimentally analyze and interpret binding data and binding curves.

Hackert – CH370

Goals for this Unit

- **Understand basic ligand binding equation**
 - essential terms and equations
 - equilibrium binding / meaning of K_d / van’t Hoff plots
 - When you can simply by assuming $[S] \sim [S_0]$
- **Complex equilibrium binding**
 - Multiple sites / independent or cooperative
 - Diff. Microscopic vs. Macroscopic binding constants
 - Scatchard plots and Hill Plots
- **Techniques to determine K_d**
 - Simple (Equil. Dialysis; Fluor) / ITC / SPR
 - How to derive K_d from Equil. Dialysis data
 - How to interpret Fluor / ITC and SPR data

Summary of Key Equations / Relationships



$$K_d = k_{\text{off}} / k_{\text{on}} = [E][S]/[ES] \quad \text{and} \quad K_a = 1 / K_d$$

$$S_o = S + ES; \quad E_o = E + ES$$

define Fractional Occupancy of sites

$$q = [ES]/[E_o] = [ES]/([E] + [ES]) \times [S]/[S] \times [1/ES]/[1/ES]$$

$$\text{then} \quad q = [S]/(K_d + [S])$$

$$\text{thus when } [S] = K_d, \text{ then } q = 0.50$$

$$\text{when } [S] = 4K_d, \text{ then } q = 0.80$$

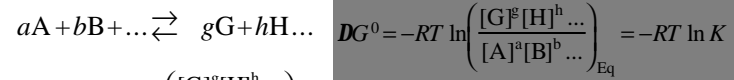
$$\text{when } [S] = 10K_d, \text{ then } q = 0.91$$

Note: $[S]$ = conc. of free ligand!!

What is the meaning of the dissociation constant (K_d) for binding of a single ligand to its site?

1. K_d has units of concentration, M or mol/liter
2. K_d gives the concentration of ligand to saturate 50% of the ligand binding sites (when the total site concentration is much lower than K_d)
3. Almost all binding sites are saturated when the free ligand concentration is $10 \times K_d$
4. The dissociation constant K_d is related to Gibbs free energy ΔG° by the relation $\Delta G^\circ = -RT \ln K_d$

?G, ?G° of an reaction at equilibrium



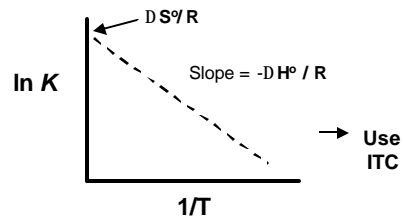
$$0 = DG^0 + RT \ln \left(\frac{[G]^g [H]^h \dots}{[A]^a [B]^b \dots} \right)_{Eq}$$

$$K = \left(\frac{[G]^g [H]^h \dots}{[A]^a [B]^b \dots} \right)_{Eq} = \exp \left(\frac{-\Delta G^0}{RT} \right)$$

$$\underline{DG^0} = \underline{DH^0} - T \underline{DS^0}$$

van't Hoff Equation

$$\ln K = \frac{-\underline{DH^0}}{RT} + \frac{\underline{DS^0}}{R}$$



K_d values in biological systems

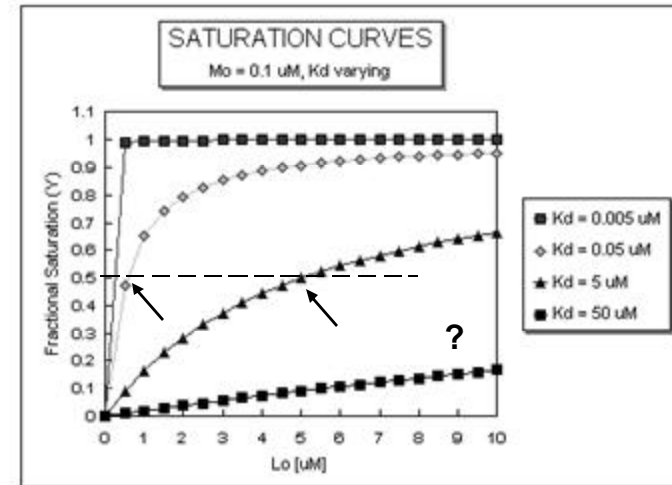
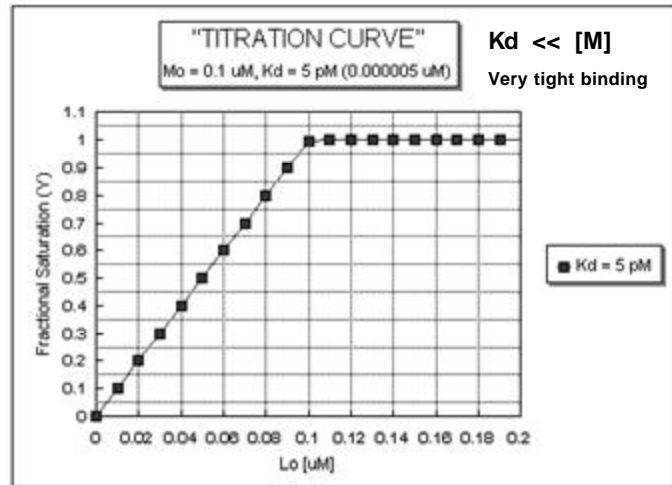
Movivalent ions binding to proteins or DNA have K_d 0.1 mM to 10 mM

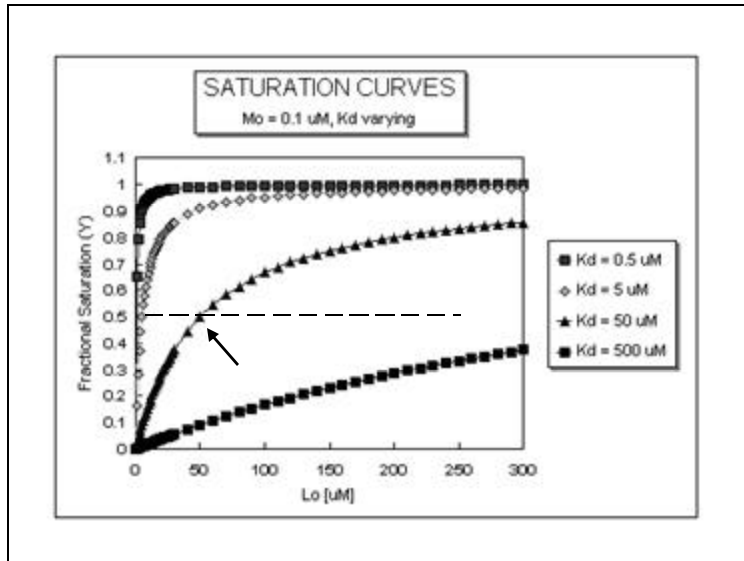
Allosteric activators of enzymes e. g. NAD have K_d 0.1 μM to 0.1 mM

Site specific binding to DNA K_d 1 nM to 1 pM

Trypsin inhibitor to pancreatic trypsin protease K_d 0.01 pM

Antibody-antigen interaction have K_d 0.1 mM to 0.0001 pM





Simplification of Key Equations

$E + S \rightleftharpoons ES$; for single site

$K_d = k_{off} / k_{on} = [E][S]/[ES]$ and $K_a = 1 / K_d$

$S_o = S + ES$; $E_o = E + ES$

If $S_o \gg E_o$, then $S \sim S_o$

then $K_d [ES] = [E][S] = [E_o - ES][S_o]$

$[ES] = E_o S_o / (K_d + S_o)$;

define Fractional Occupancy of sites

→ $q = [ES]/[E_o] = [S_o]/(K_d + [S_o])$

thus when $[S_o] = K_d$, then $q = 0.5$

No Assumptions - Key Equations

Derivation of quadratic equation--with no assumptions concerning substrate concentration.

$fraction = \theta = [ES]/[E]_o = [ES]/([E] + [ES])$ The derivation starts the same as above

$\theta = \frac{K_a[E][S]}{[E] + K_a[E][S]} = \frac{K_a[S]}{1 + K_a[S]} = \frac{[S]}{1/K_a + [S]}$ Fraction of sites bound relative to [S]

$\theta = \frac{[S]}{K_d + [S]} = \frac{[S]_o - [ES]}{K_d + [S]_o - [ES]} = \frac{[ES]}{[E]_o}$ Substitution of [S]=[S]_o-[ES]

$[ES](K_d + [S]_o) - [ES]^2 = [E]_o[S]_o - [ES][E]_o$

$[ES]^2 - [ES](K_d + [S]_o + [E]_o) + [E]_o[S]_o = 0$ Form of equation require solution as the roots of the quadratic equation

No Assumptions - Key Equations

Solution is quadratic equation:

$ax^2 + bx + c = 0$

solution provided by the roots of the quadratic

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$[ES] = \frac{(E_o + S_o + K_d) - \sqrt{(E_o + S_o + K_d)^2 - 4 \cdot E_o \cdot S_o}}{2}$

→ $\theta = \frac{(E_o + S_o + K_d) - \sqrt{(E_o + S_o + K_d)^2 - 4 \cdot E_o \cdot S_o}}{2 \cdot E_o}$

This equation must be used rather than the hyperbola whenever the enzyme concentration is comparable or greater than the dissociation constant. As a general rule, if $[E]_o$ is less than 5 times the K_d , the hyperbolic fit is probably adequate.

Manipulations of Equations

a) double reciprocal plot

$$1/q = K_d/[S] + 1; \text{ plot } 1/q \text{ vs. } 1/[S]$$

b) Scatchard Plot: $q = [S]/(K_d + [S])$ or

$$qK_d + q[S] = [S] \text{ or } q = 1 - qK_d/[S]$$

$$\text{plot } q \text{ vs. } q/[S] \text{ slope} = -K_d$$

Linearized forms of the equation:

a) Double Reciprocal Plot

$$1/\theta = \frac{[S] + K_d}{[S]} = 1 + \frac{K_d}{[S]}$$

Or for multiple sites:

$$1/\nu = \frac{[S] + K_d}{[S]} = n + \frac{K_d}{[S]}$$

b) Scatchard Plot

$$\theta = 1 - \frac{\theta K_d}{[S]}$$

Or for multiple sites:

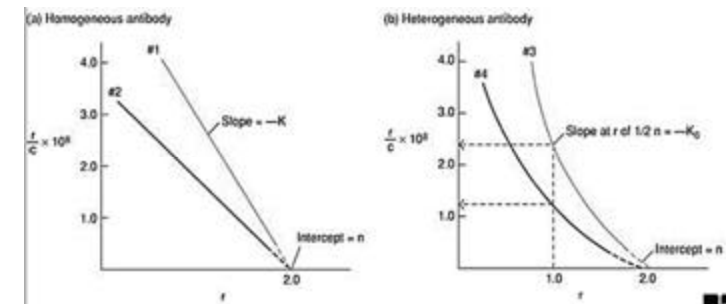
$$\theta = n - \frac{\nu K_d}{[S]}$$

$$\nu = \frac{\text{moles bound}}{\text{mole E}} = n\theta$$

Scatchard equation: $r/c = K(n-r)$

Scatchard plot - determine the equilibrium association constant and valence of an antibody (Historical – useful for visualizing data, but not the most accurate way to analyze the data! Use nonlinear regression to fit data).

- r = moles bound ligand/mole antibody at equilibrium;
- c = free ligand concentration at equilibrium;
- K = equilibrium association constant; and
- n = number of antigen binding sites per antibody molecule



EXPERIMENTAL DETERMINATION OF K_d

TECHNIQUES THAT REQUIRE SEPARATION OF BOUND FROM FREE LIGAND – Care must be given to ensure that the equilibrium of $M + L \rightleftharpoons ML$ is not shifted during the separation technique.

• **gel filtration chromatography** - Add M to a given concentration of L. Then elute the mixture on a gel filtration column, eluting with the free ligand at the same concentration. The ML complex will elute first and can be quantified. If you measure the free ligand coming off the column, it will be constant after the ML elutes with the exception of a single dip near where the free L would elute if the column was eluted without free L in the buffer solution. This dip represents the amount of ligand bound by M.

• **membrane filtration** - Add M to radiolabeled L, equilibrate, and then filter through a filter which binds M and ML. For instance, a nitrocellulose membrane binds proteins irreversibly. Determine the amount of radiolabeled L on the membrane which equals [ML].

• **precipitation** - Add a precipitating agent like ammonium sulfate, which precipitates proteins and hence both M and ML. Determine the amount of ML.

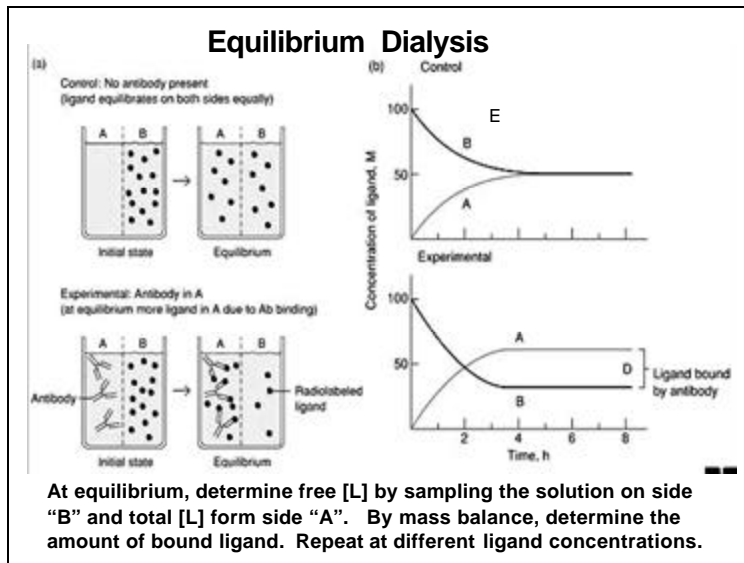
EXPERIMENTAL DETERMINATION OF K_d

TECH. THAT DO NOT REQUIRE SEPARATION OF BOUND FROM FREE LIGAND

• **equilibrium dialysis** - Place M in a dialysis bag and dialyze against a solution containing a ligand whose concentration can be determined using radioisotopic or spectroscopic techniques. At equilibrium, determine free L by sampling the solution surrounding the bag. By mass balance, determine the amount of bound ligand, which for a 1:1 stoichiometry gives ML. Repeat at different ligand concentrations.

• **gel shift assay** – Use autoradiography and gel electrophoresis with radiolabeled macromolecule.

• **spectroscopy** - Find a ligand whose absorbance or fluorescence spectra changes when bound to M. Alternatively, monitor a group on M whose absorbance or fluorescence spectra changes when bound to L.

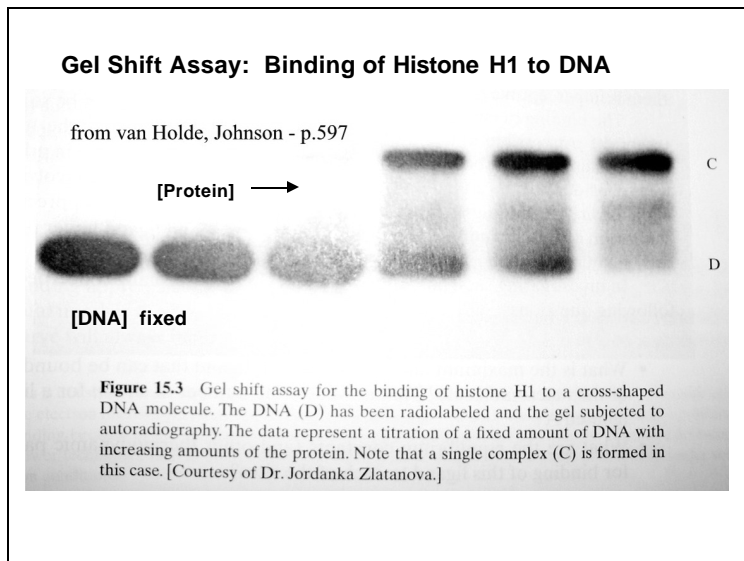


Multi-Equilibrium Dialyzer™

The Harvard Apparatus Multi-Equilibrium Dialyzer provides highly standardized equilibrium dialysis conditions for up to 20 parallel assays. The instrument offers outstanding sensitivity of:

- Membrane Area
- Sample Volume
- Degree of Agitation

The dialyzer cells are made of Teflon, an extremely inert material, and will not interfere with the samples. Multiple cell systems are available (5, 10, 15, 20 cells) at various cell volumes (0.25, 1.0, 2.0 & 5.0ml). The unit can be sterilized by autoclaving and the cells can be filled easily with a filling clamp.



Spectroscopy

Fluorescence Spectroscopy

$F = F_0 + \Delta F \cdot \theta$ This equation normalizes the observable signal to a scale that can be related to fractional occupancy, θ .

where $\Delta F = F_{\infty} - F_0$

and θ is defined by either:

$$\theta = \frac{[S]_0}{K_d + [S]_0} \quad \text{OR} \dots \quad \theta = \frac{(E_0 + S_0 + K_d) - \sqrt{(E_0 + S_0 + K_d)^2 - 4 \cdot E_0 \cdot S_0}}{2 \cdot E_0}$$

Fluorescence Anisotropy

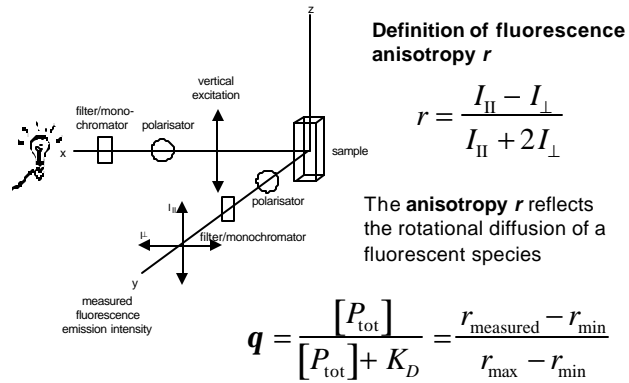
Definition of fluorescence anisotropy r

$$r = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + 2I_{\perp}}$$

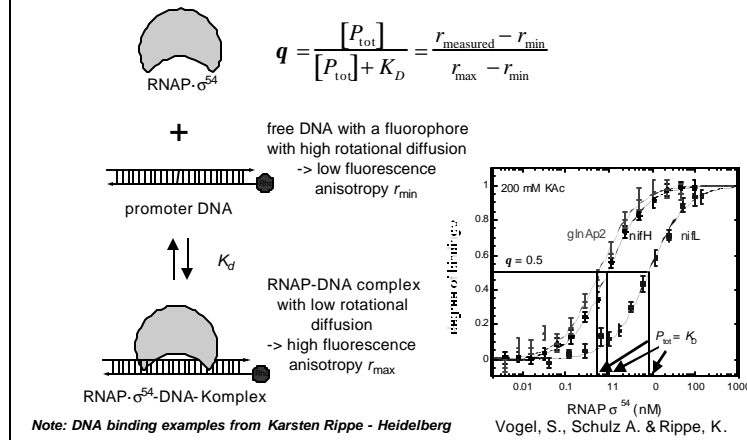
$$q = \frac{F - F_0}{\Delta F = F_{\infty} - F_0}$$

$$q = \frac{r_{\text{measured}} - r_{\text{min}}}{r_{\text{max}} - r_{\text{min}}}$$

**How to measure binding of a protein to DNA?
One possibility is to use fluorescence anisotropy**

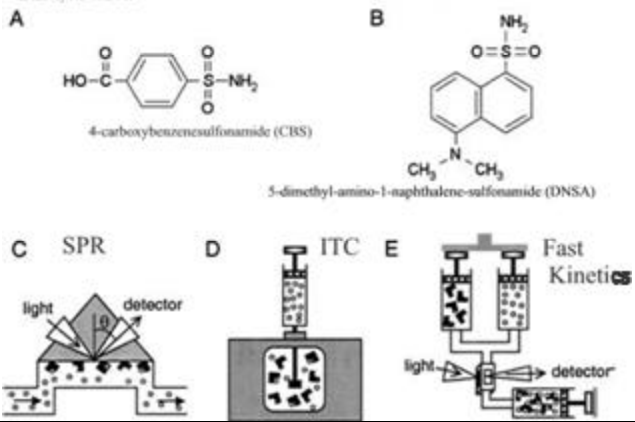


Analysis of binding of RNAP- σ^{54} to a promoter DNA sequence by measurements of fluorescence anisotropy

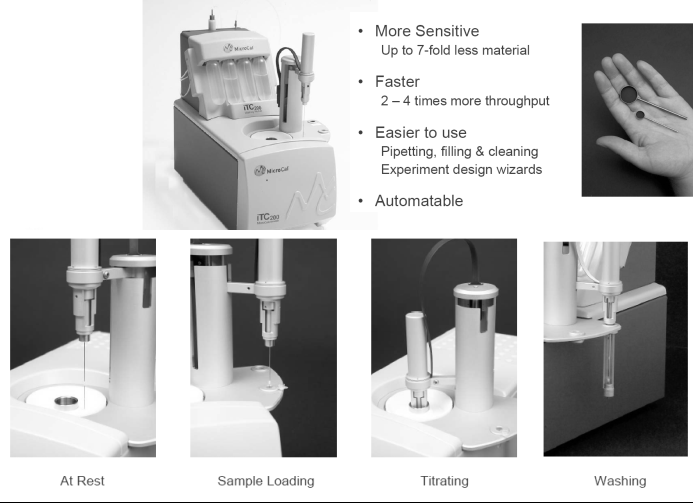


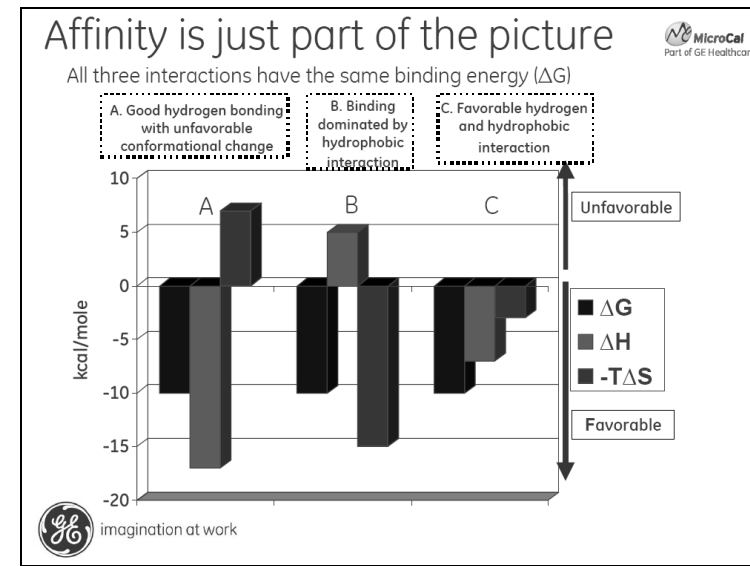
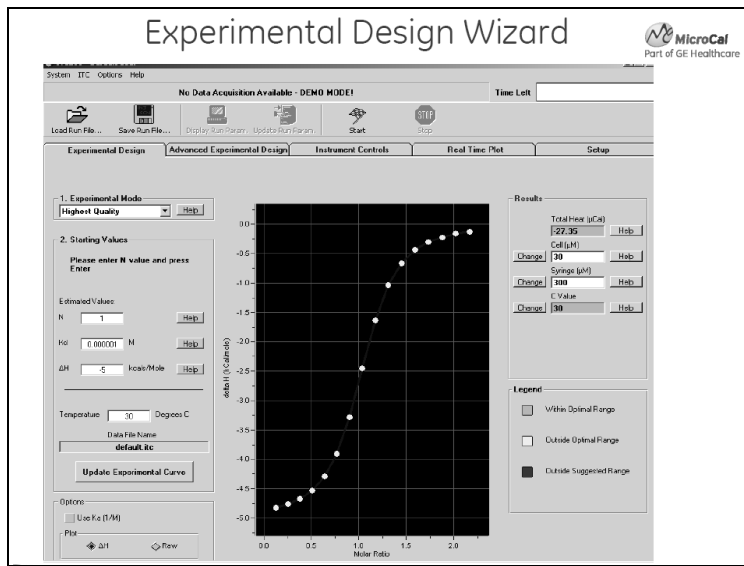
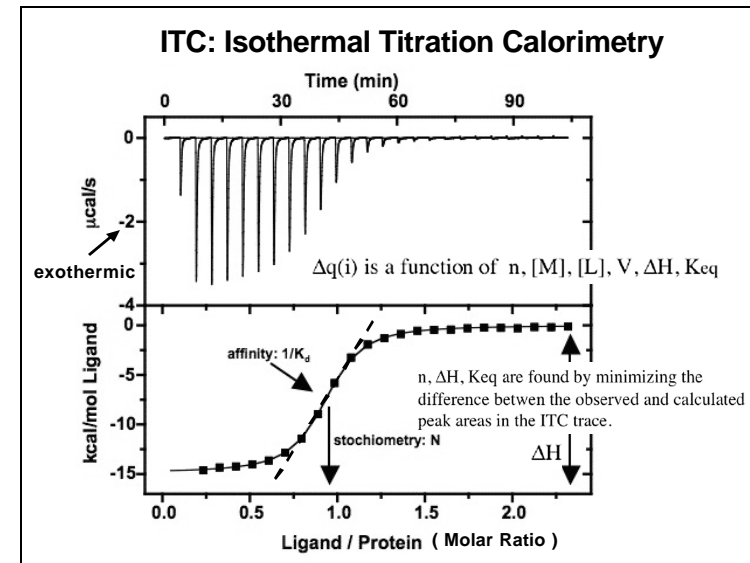
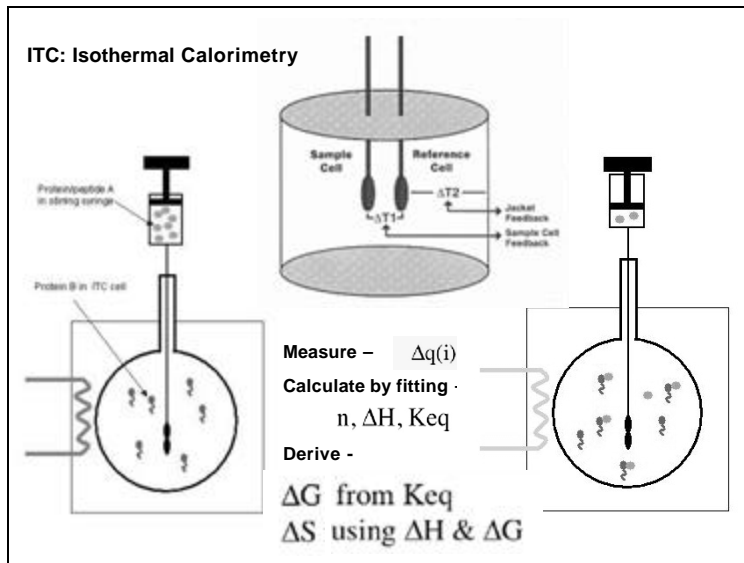
Direct comparison of binding equilibrium, thermodynamic, and rate constants determined by surface- and solution-based biophysical methods
(Binding of Small Molecules CBS & DNSA to Carbonic Anhydrase II)

YASMINA S.N. DAY, CHERYL L. BAIRD, REBECCA L. REIL, and DAVID G. HYSZKA
Center for Biomolecular Simulation Analysis, University of Utah, School of Medicine, Salt Lake City, Utah 84143, USA



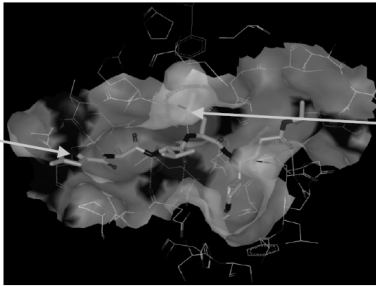
The iTC₂₀₀





Microcalorimetry provides a total picture of binding energetics

Overall binding affinity K_D correlates with IC_{50} or EC_{50} . This is directly related to ΔG the total free binding energy



ΔH is a measure of hydrogen and van der Waals bonding

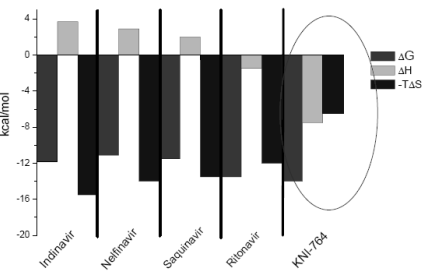
$-T\Delta S$ is a measure of hydrophobic interaction and conformational changes

n = Stoichiometry indicates the ratio of ligand molecules that bind each macromolecule

MicroCal Part of GE Healthcare

GE imagination at work

Drug Discovery – Similar Affinities... Different Mechanisms



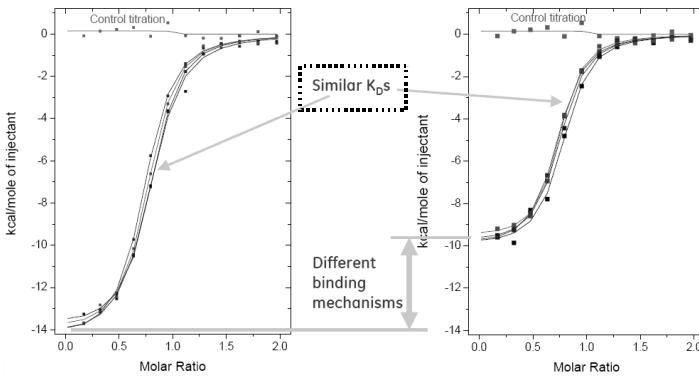
Inhibitor	ΔG	ΔH	$-T\Delta S$
Indinavir	-12	-1	-1
Nelfinavir	-14	-1	-1
Saquinavir	-13	-1	-1
Ritonavir	-13	-1	-1
K11-764	-10	-1	-1

- Binding of inhibitors to HIV-1 protease.
- Favorable enthalpy and entropy is the best profile for drug-target interaction

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GE imagination at work

Carbonic anhydrase-inhibitor titrations with Auto-iTC₂₀₀



CBS titrations

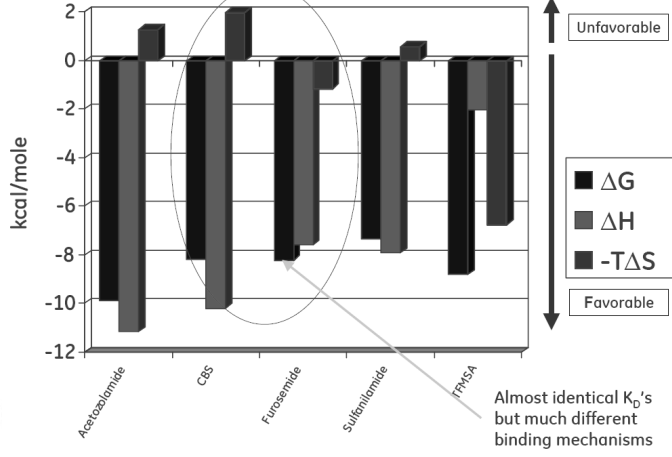
Furosemide titrations

Similar K_D s

Different binding mechanisms

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Carbonic anhydrase inhibitors



Inhibitor	ΔG	ΔH	$-T\Delta S$
Acetazolamide	-10	-11	1
CBS	-10	-10	1
Furosemide	-8	-8	0
Sulfanilamide	-7	-8	1
FX34	-9	-9	0

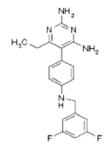
Unfavorable

Favorable

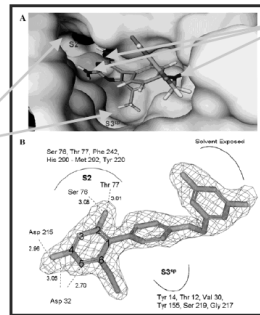
Almost identical K_D 's but much different binding mechanisms

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The Binding Orientation for Lead Template to Renin was Determined X-ray Crystallography

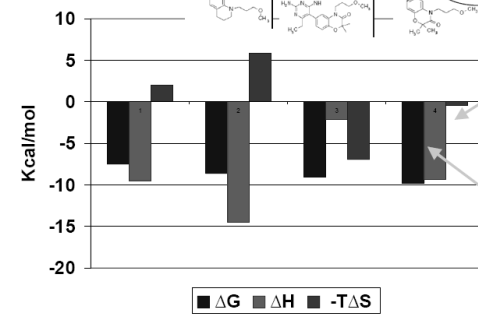
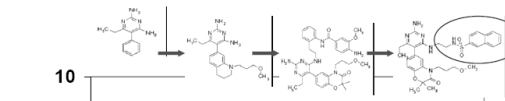


The unoccupied hydrophobic S2 and S3 pockets are opportunities to enhance favorable ΔS



favorable ΔH is consistent with the strong network of hydrogen bonds.

Modeling Suggests Substituting Aryl-Benzamide with Aryl-Sulfonamide to Improve H-bonds



Favorable $-T\Delta S$ due to conversion of hydrophobic binding in S2 pocket to H-bonds

Dramatic Increase in ΔH is consistent with increase in S2 pocket H-bonds

■ ΔG ■ ΔH ■ $-T\Delta S$

Another 3.4X improvement in affinity

Renin Inhibitor Affinity Improved 45X from Initial 3.6 μM Lead to 79nM

- S3 Pocket – Ether addition improved enthalpy due to van der Waals bonds
- S2 Pocket – Aryl-Sulfonamide improved binding enthalpy while retaining hydrophobic advantage



imagination at work

Summary: Isothermal Titration Calorimetry (ITC)

Good things about ITC:

Accurate determination of binding (K_d) and thermodynamic (ΔG , ΔH , ΔS) parameters for ligand-macromolecule interactions.

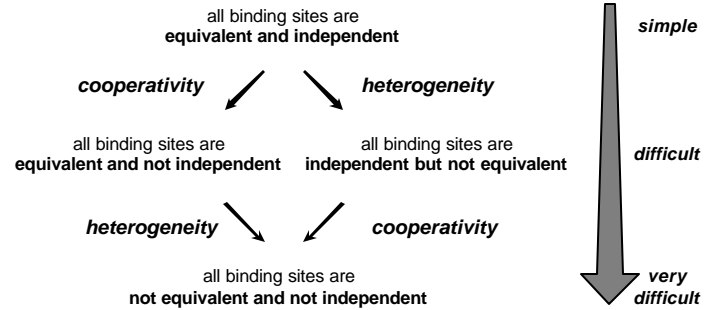
ITC does not make the approximations that are included in a van't Hoff type of analysis.

Not such good things about ITC:

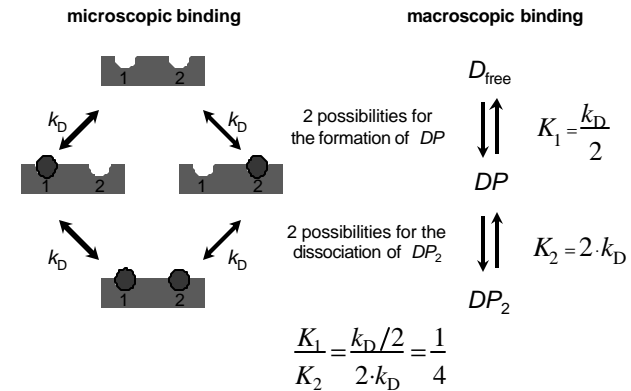
Large (perhaps 10 mg) quantities of material are required, in order to detect the small amount of heat released upon mixing macromolecule and ligand.

In comparison, gel-mobility shift assays can be carried out with $\ll 1$ mg of material.

Increasing complexity of binding



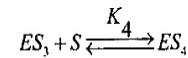
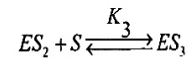
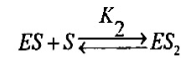
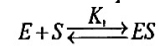
Difference between microscopic and macroscopic dissociation constant



Multiple Binding Equilibria

Multiple Binding Sites:

Reaction step



Association constant K_a

$$[ES] = K_1[E][S]$$

$$[ES_2] = K_2[ES][S] = K_1K_2[E][S]^2$$

$$[ES_3] = K_3[ES_2][S] = K_1K_2K_3[E][S]^3$$

$$[ES_4] = K_4[ES_3][S] = K_1K_2K_3K_4[E][S]^4$$

Adair Equation (1925)

Solution of fraction of sites occupied for a two-step binding sequence.

Mass balance equations:

$$[E]_0 = [E] + [ES] + [ES_2]$$

$$[S]_0 = [S] \text{ (negligible amount bound)}$$

Fraction of sites bound:

$$\theta = ([ES] + [ES_2]) / [E]_0$$

$$= \frac{[ES] + [ES_2]}{[E] + [ES] + [ES_2]}$$

Substitution of bound states:

$$[ES] = K_1[E][S]$$

$$[ES_2] = K_2[ES][S] = K_1K_2[E][S]^2$$

$$\theta = \frac{K_1[E][S] + K_1K_2[E][S]^2}{[E] + K_1[E][S] + K_1K_2[E][S]^2}$$

$$\theta = \frac{[S]_0}{K_d + [S]_0}$$

1 Site

2 Independent Sites

$$\theta = \frac{K_1[S] + K_1K_2[S]^2}{1 + K_1[S] + K_1K_2[S]^2}$$

Adair Equation (1925)