

X-Ray Crystallography

“If a picture is worth a thousand words, then a macromolecular structure is priceless to a physical biochemist.” – van Holde

Topics:

1. Image Formation (*optical illusions*)

Resolution / Wavelength (Amplitude, Phase) / Light Microscopy / EM / X-ray / (NMR)

2. Protein Data Bank (PDB)

Data mining and Protein Structure Analysis Tools

3. X-Ray Crystallography

- a) 100 years of X-ray Crystallography
- b) Crystal Growth – Materials / Methods
- c) Crystal Lattices - Lattice Constants / Space Groups / Asymmetric Unit
- d) X-ray Sources – Sealed Tube / Rotation Anode / Synchrotron
- e) Theory of Diffraction – Bragg’s Law / Reciprocal Space
- f) Data Collection – Methods / Detectors / Structure Factors
- g) Structure Solution – Phase Problem: MIR / MR / MAD
- h) Refinements and Models / Analysis and presentation of results

Object



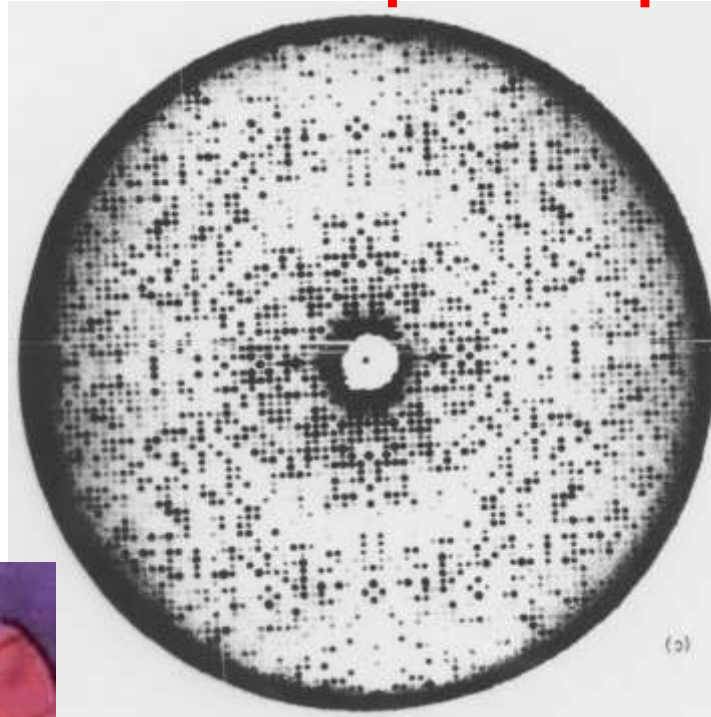
Transform



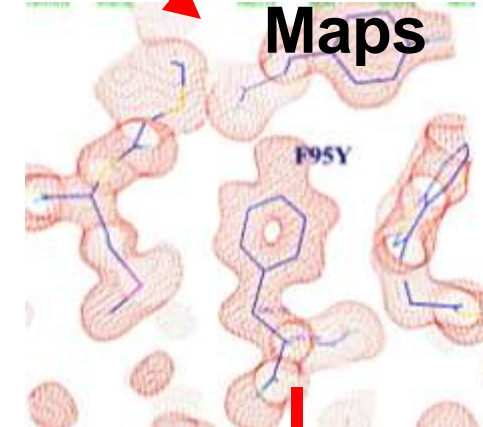
Image

Transform / Reciprocal Space

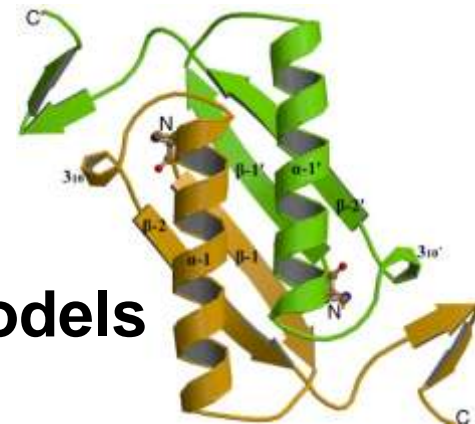
Object / Real Space



Electron
Density
Maps



Models



IYCr 2014



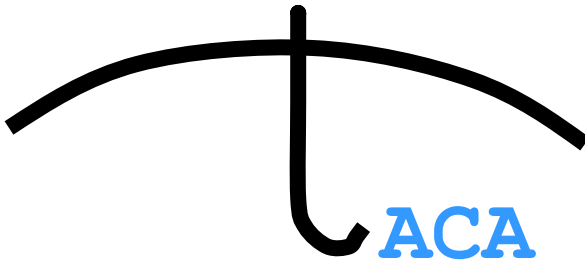
Marv Hackert - UT Austin - Fall 2013

<http://www.theguardian.com/science/video/2013/oct/09/100-years-x-ray-crystallography-video-animation>

Relational Chart

ACA / AIP / USNCCr / IUCr / ICSU

AIP



ACA Council

12 SIGs / ~ 1600 Members

(1 of

				
Acoustical Society of America	American Association of Physicians in Medicine	American Association of Physics Teachers	American Astronomical Society	Optical Society of America
				
American Crystallographic Association	American Geophysical Union	American Physical Society	AVS: Science & Technology of Materials, Interfaces, and Processing	The Society of Rheology

30 Intl.
Unions

ICSU

52
Members

IUCr ~24
Commissions

(1 of 3)

(1 of ~52)

(NAS)

ACA
ECA
AsCA

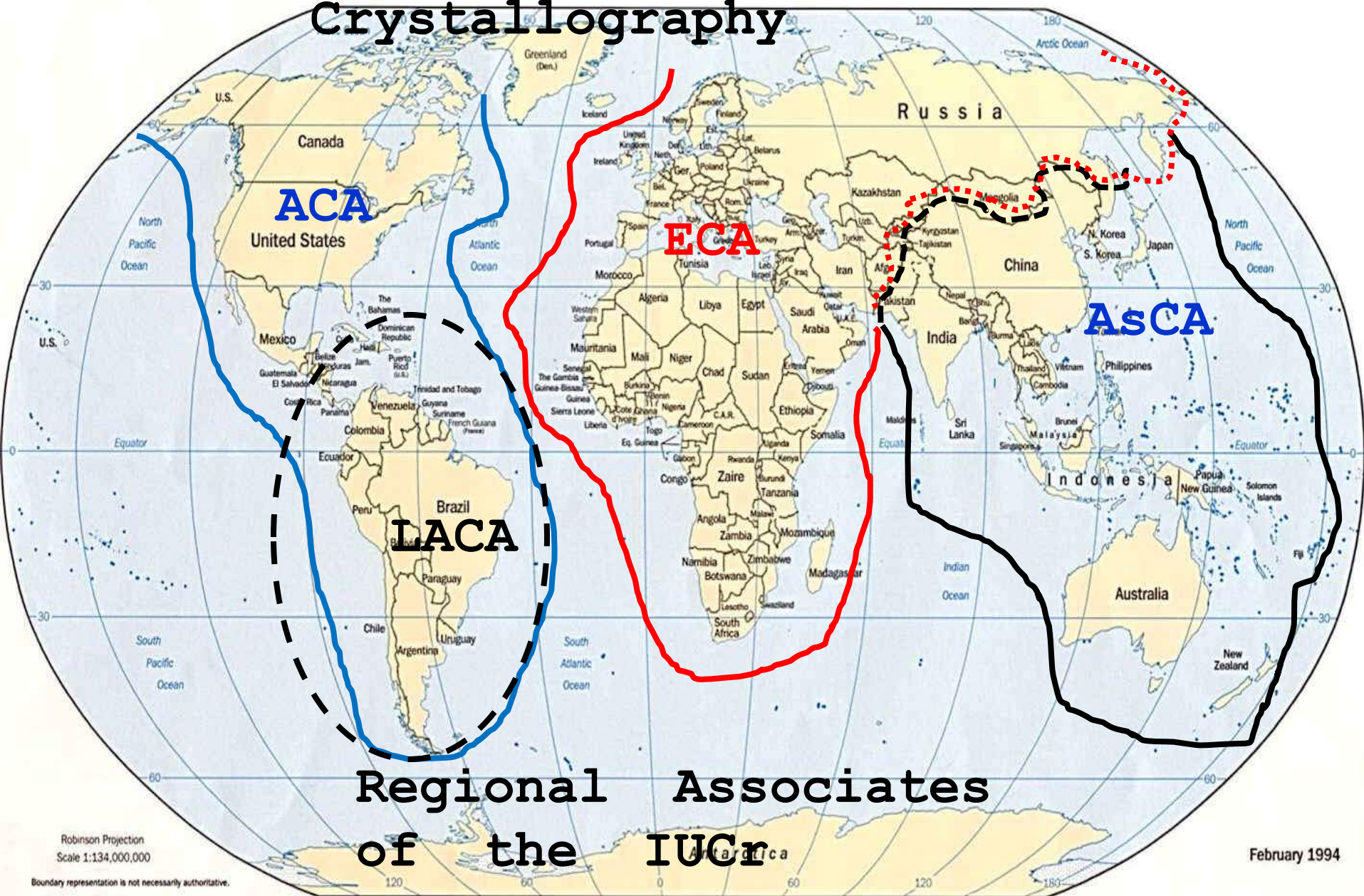
USNC/Cr

IUCN
r



Countries that adhere to the IUCr are shown in red (43 Adhering Bodies of the IUCr, representing 52 countries, and its **three Regional Associates**).

The World of Crystallography

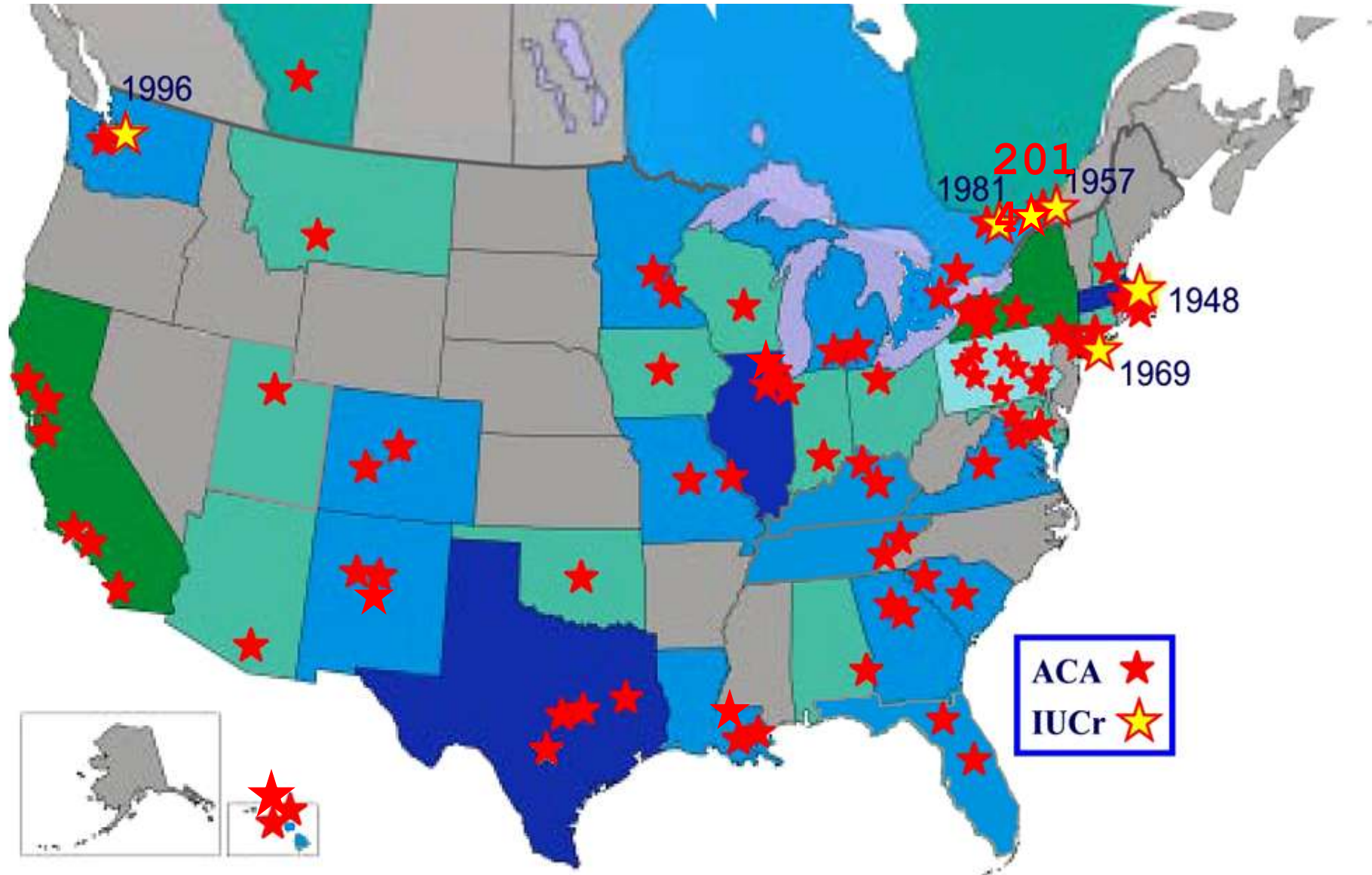


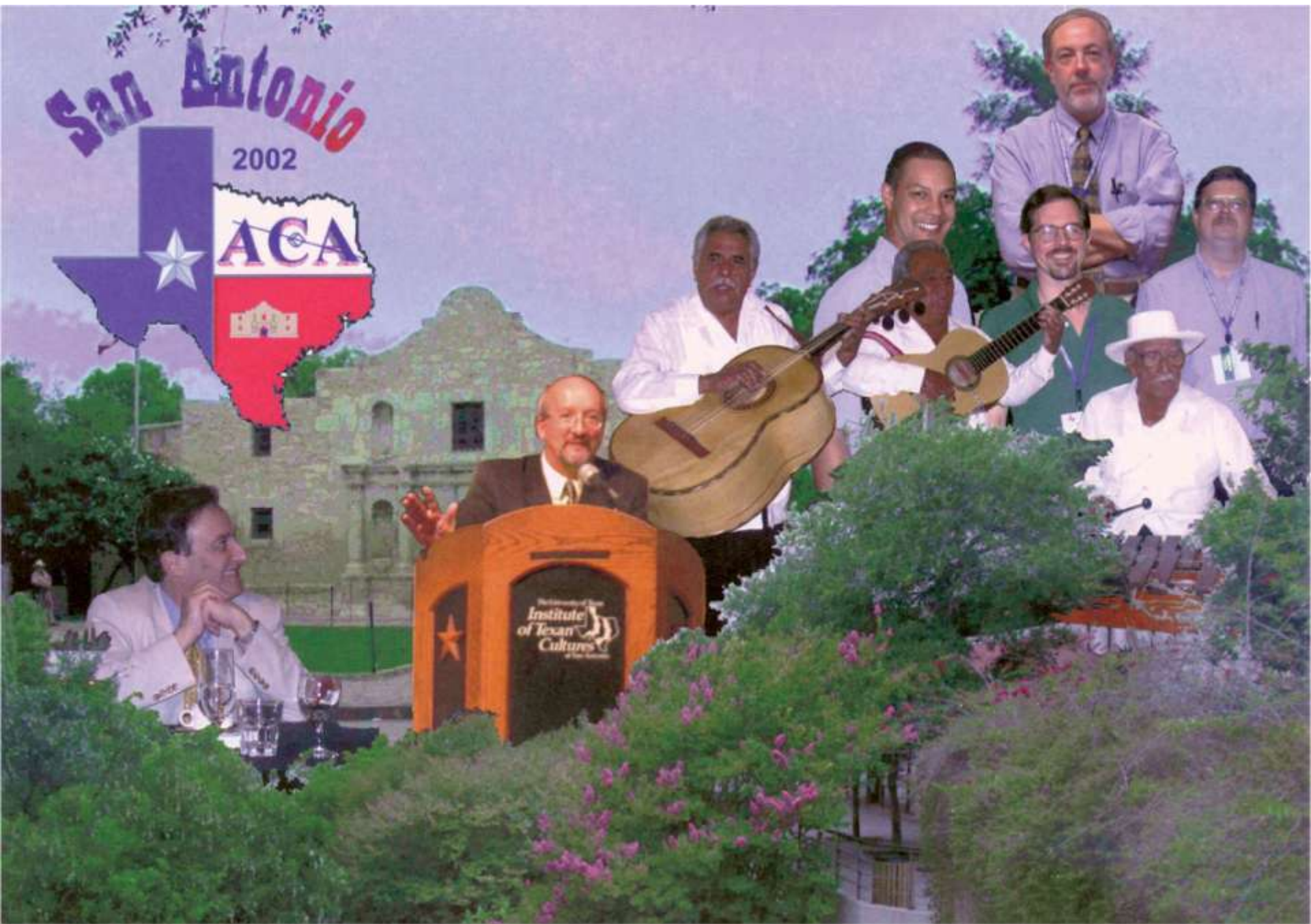
Robinson Projection
Scale 1:134,000,000

Boundary representation is not necessarily authoritative.

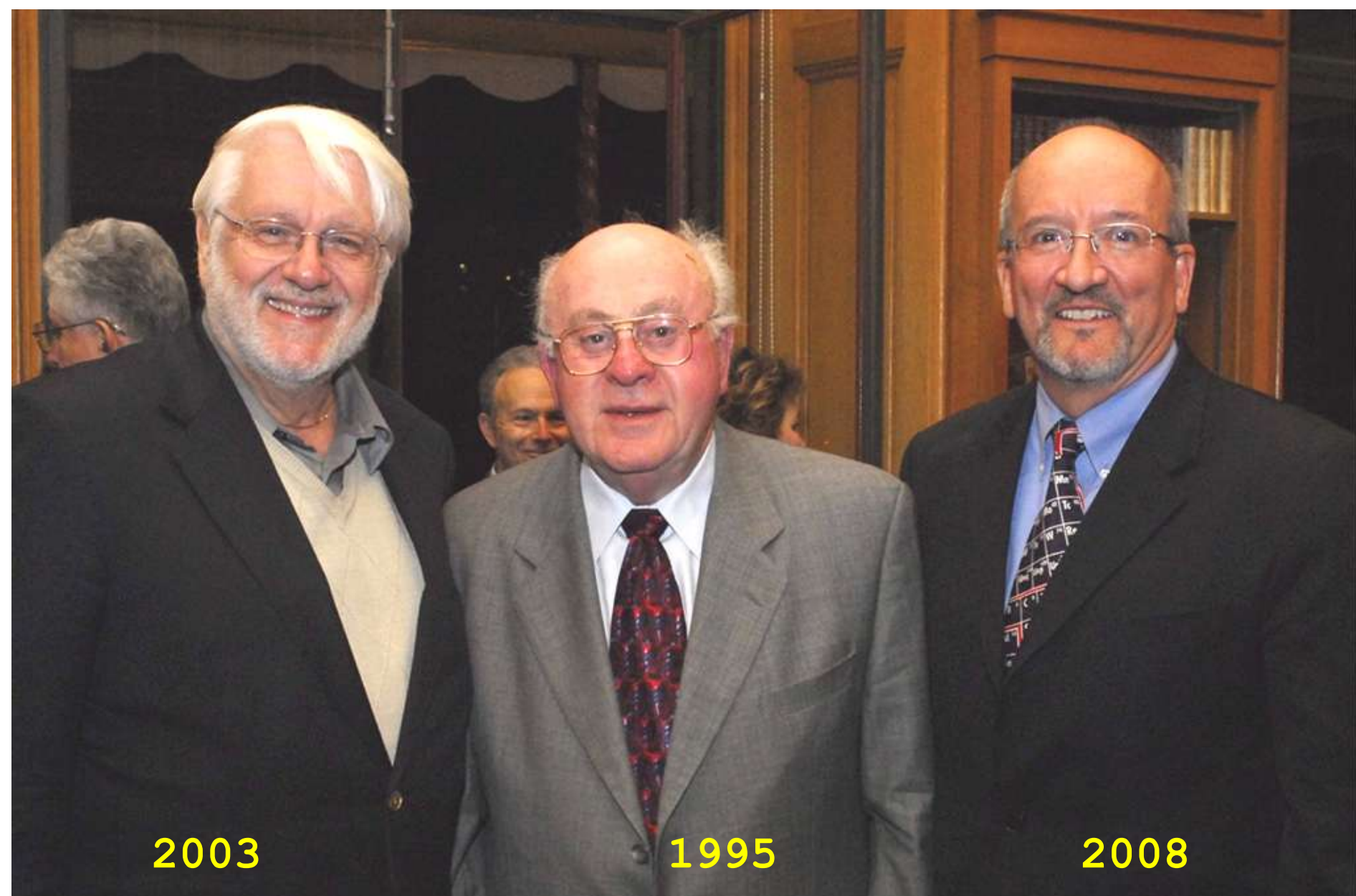
February 1994

American Crystallographic Assn.





University of Texas at Austin – ACA Presidents





Front Row: Elena Boldyreva, Claude Lecomte, Gautam Desiraju, Luc Van Meervelt, Sine Larsen; **Back Row:** Wulf Depmeier, Michael Dacombe, Hanna Dabkowska, Marvin Hackert, Mitchell Guss, Manuel Perez-Mato.

Bragg Symposium: Celebrating 100 years of X-ray crystallography - *The University of Adelaide, 6th December 2012*



Celebrating 100 Years of X-Ray Crystallography



Photos of Lawrence Bragg courtesy of Dr S L Bragg.
Image of Mitchell Building at University of Adelaide,
courtesy of the University of Adelaide Archives.


Australian Academy of Science



opening ceremony
unesco building
paris
20-21 january 2014



IYCr2014 Opening Ceremony ADMISSION TICKET

Paris – UNESCO Headquarters –

Room 1

7 Place de Fontenoy, 75007 Paris

20-21 January 2014



Partners for the International Year of Crystallography 2014



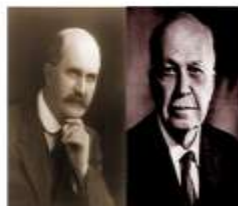
2014

international year of

Nobel Prize in Physics 1915

Sir William Henry Bragg and Sir
William Lawrence Bragg

for their services in the analysis of crystal
structure by means of X-rays

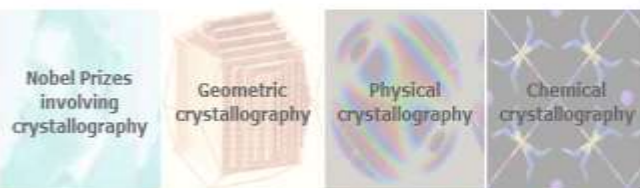


More... [W.H. Bragg](#) | [W.L. Bragg](#)

... was awarded the Physics Prize for the discovery of the diffraction of X-rays by crystals. Other
... highlight specific entries.

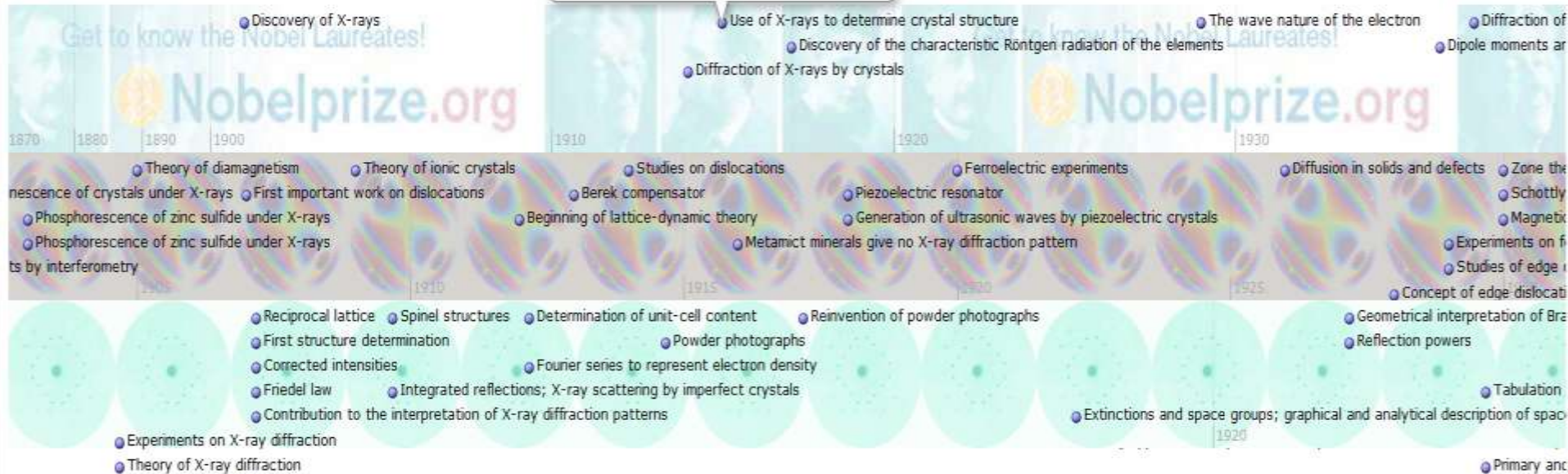
Select a date or drag the timelines.

1500	1600	1700	1750
1800	1850	1900	1950
1975	2000	2010	2014



Filter:

Highlight:

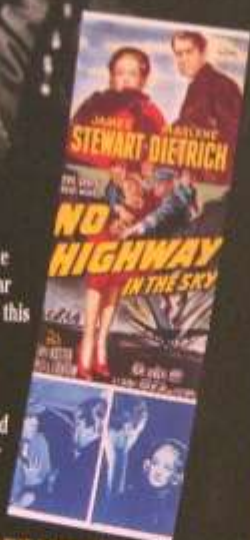


...dramatic.

ALV



...NE DIETRICH go behind the scenes of
...ing, suspense-packed classic about one
...he air. Based on Nevil Shute's popular
...olds plenty of humor and romance to this
...week)
...Honey (STEWART) has no proof, but is
...Reindeer planes will fall apart after 1440
...finds himself on a Reindeer that has reached
...warning rings true for a movie star passenger
... (GLYNIS JOHNS) who later support his
...ground the aircraft. 1951. B&W, 86 minutes.



Fox presents JAMES STEWART MARLENE DIETRICH
THE SKY GLYNIS JOHNS JACK HAWKINS JANETTE SCOTT
OSCAR MILLARD ALEC COPPEL. Based on the Novel by NEVIL SHUTE
LIGHTON Directed by HENRY KOSTER

NOT RATED



No Highway in the Sky

JAMES STEWART MARLENE DIETRICH

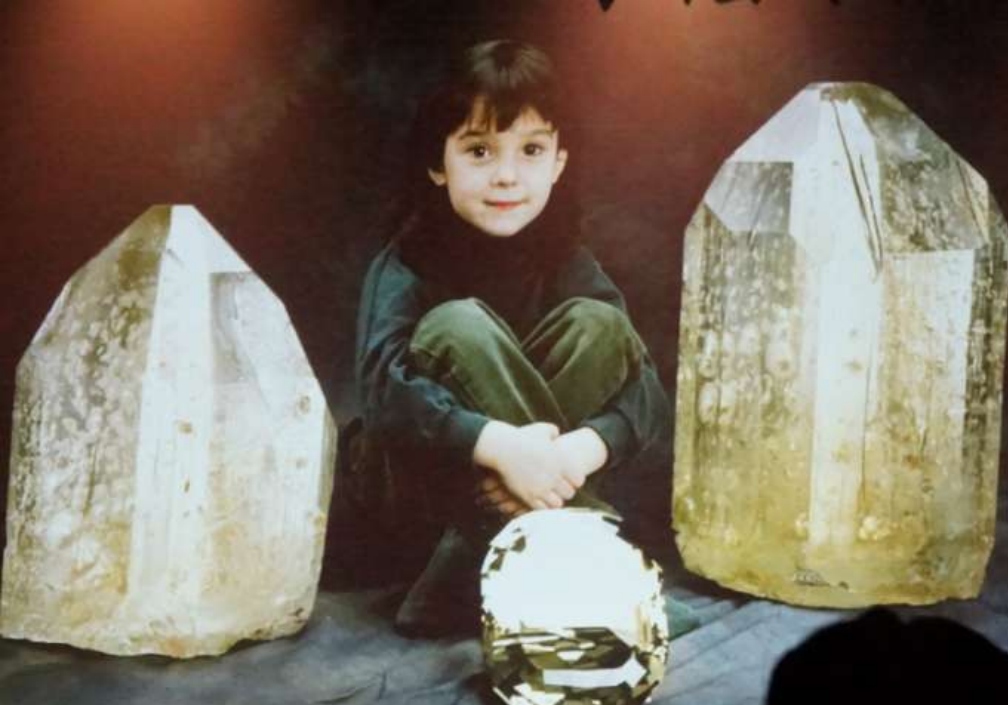
No Highway in the Sky





Crystallography: 1) the branch of science dealing with the formation and properties of crystals, 2) the science of condensed matter with emphasis on the atomic or molecular structure and its relation to physical and chemical properties

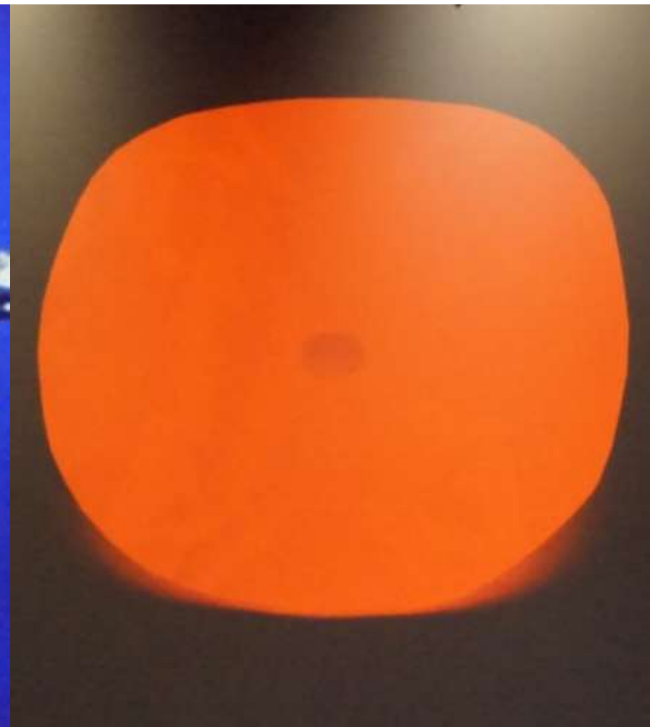




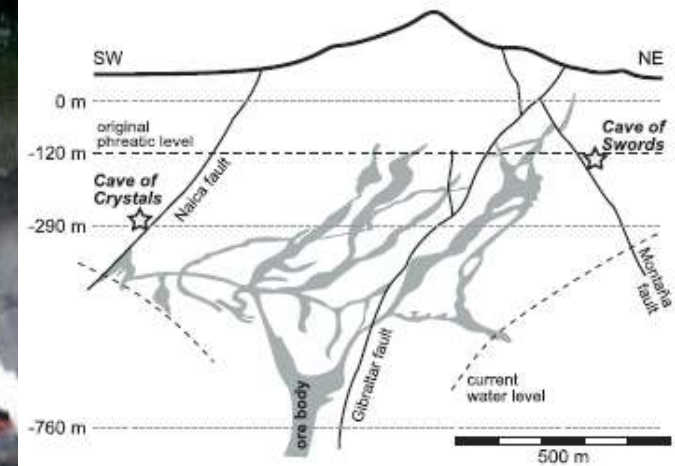
Hope Diamond



45.52 carats



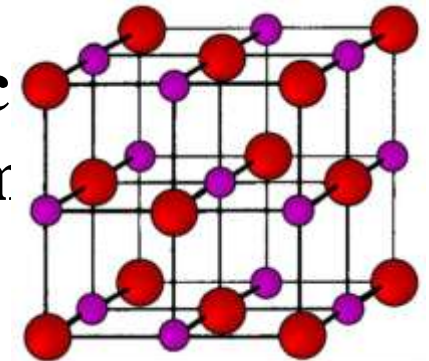
Niaca Mine, Chihuahua, Mexico



X-ray diffraction

1912 Max von Laue discovered X-ray diffraction by crystals when he and his assistants directed a beam of X-rays at a crystal of copper sulfate and record the diffraction pattern on a piece of film.

1913 W.L. Bragg reported the first crystal structure, the structure of Sodium



*Sodium Chloride
(NaCl)*



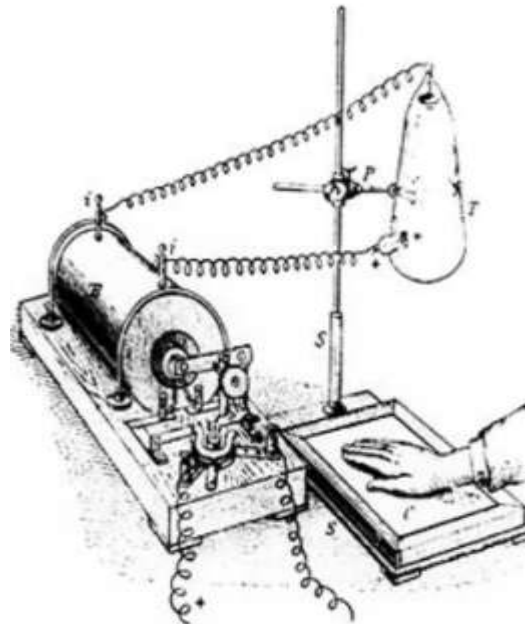
The Nobel Prize in Physics 1901
Wilhelm Conrad Röntgen



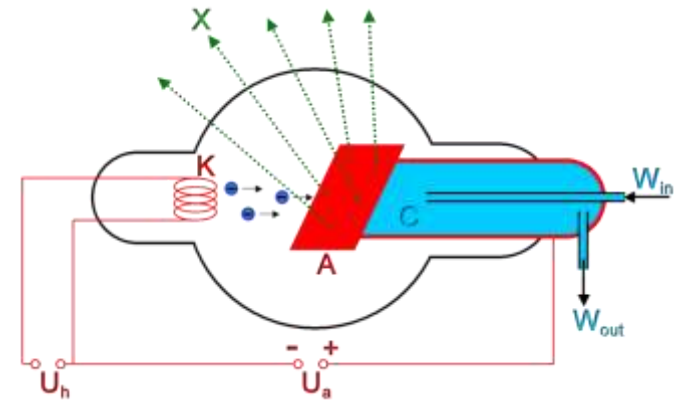
The Nobel Prize in Physics 1901



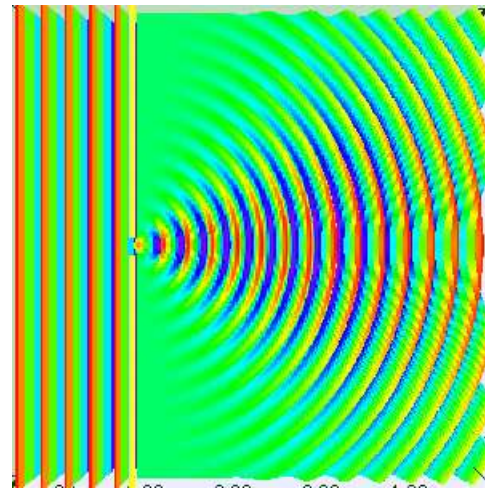
Wilhelm Conrad Röntgen



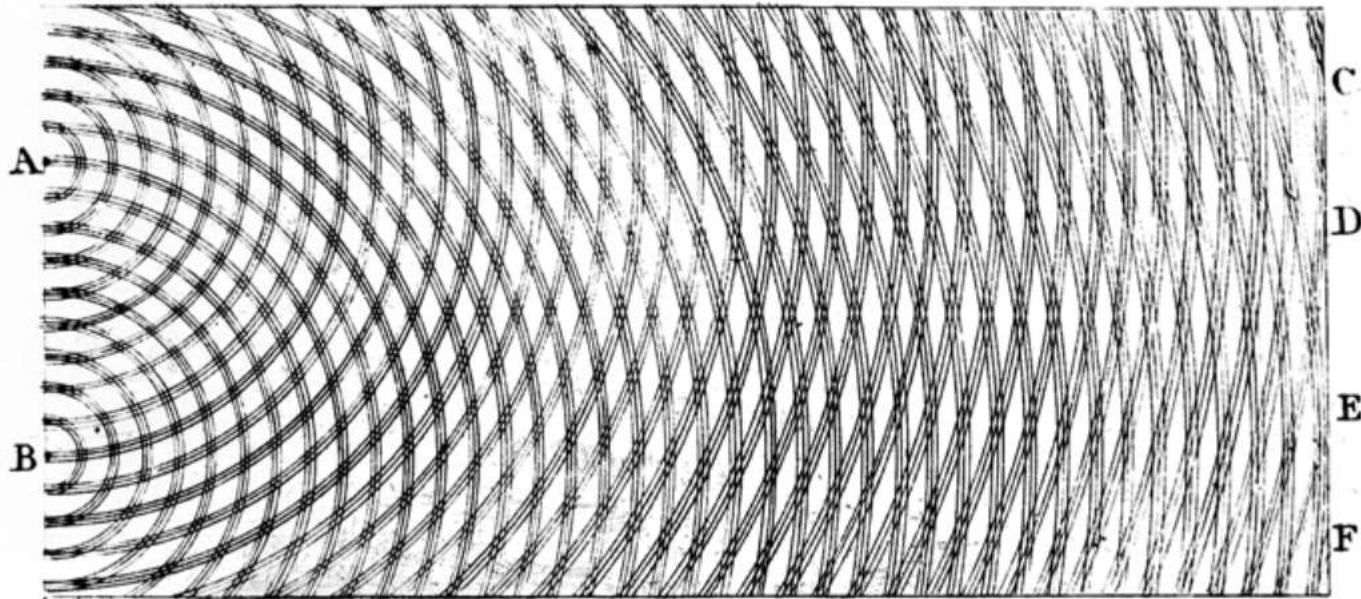
The Nobel Prize in Physics 1901 was awarded to Wilhelm Conrad Röntgen
"in recognition of the extraordinary services he has rendered by the discovery of the remarkable rays subsequently named after him".



Diffraction



Approximation of diffraction pattern from a slit of width equal to wavelength of an incident plane wave in 3D spectrum visualization.



Thomas Young's sketch of two-slit diffraction, which he presented to the Royal Society in 1803



The Nobel Prize in Physics 1914

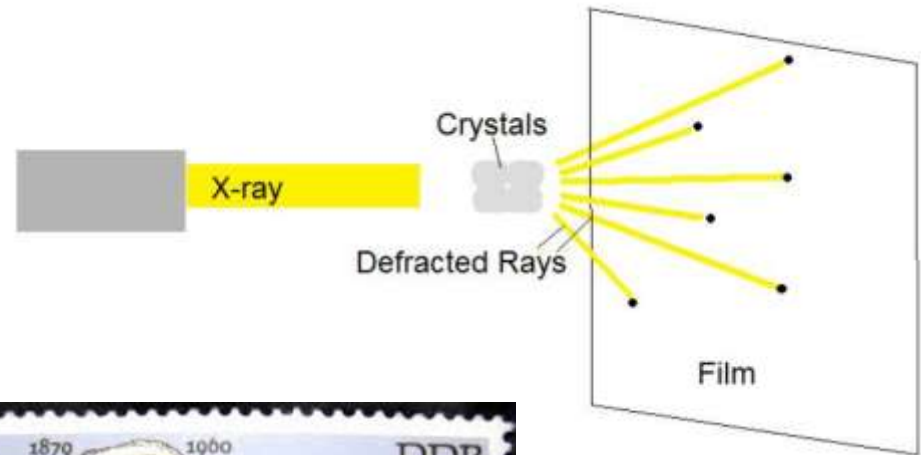
Max von Laue



The Nobel Prize in Physics 1914



Max von Laue



The Nobel Prize in Physics 1914 was awarded to Max von Laue *"for his discovery of the diffraction of X-rays by crystals"*.

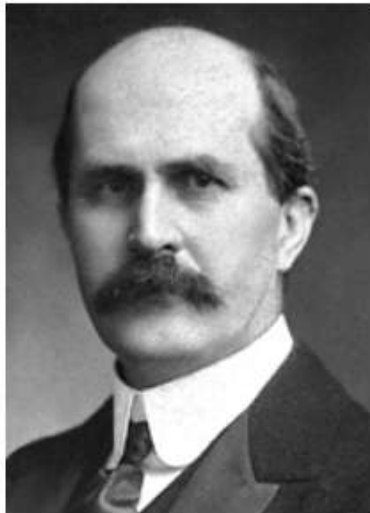


The Nobel Prize in Physics 1915

William Bragg, Lawrence Bragg



The Nobel Prize in Physics 1915



Sir William Henry Bragg

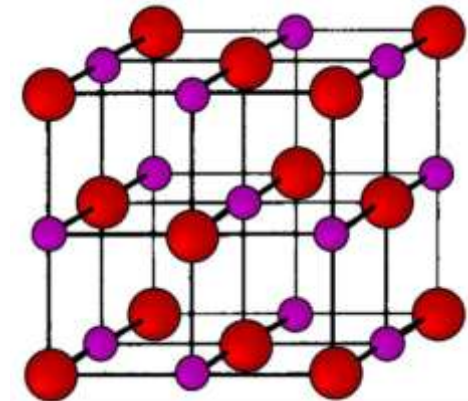


William Lawrence Bragg

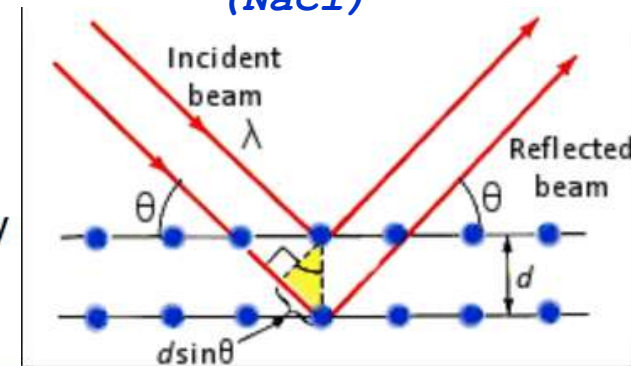
The Nobel Prize in Physics 1915 was awarded jointly to Sir William Henry Bragg and William Lawrence Bragg *"for their services in the analysis of crystal structure by means of X-rays"*



X-ray apparatus



Sodium Chloride (NaCl)



Bragg's Law ($n\lambda = 2d \sin\theta$)

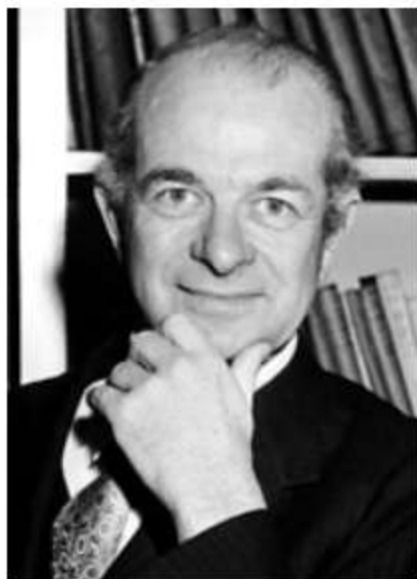


The Nobel Prize in Chemistry 1954

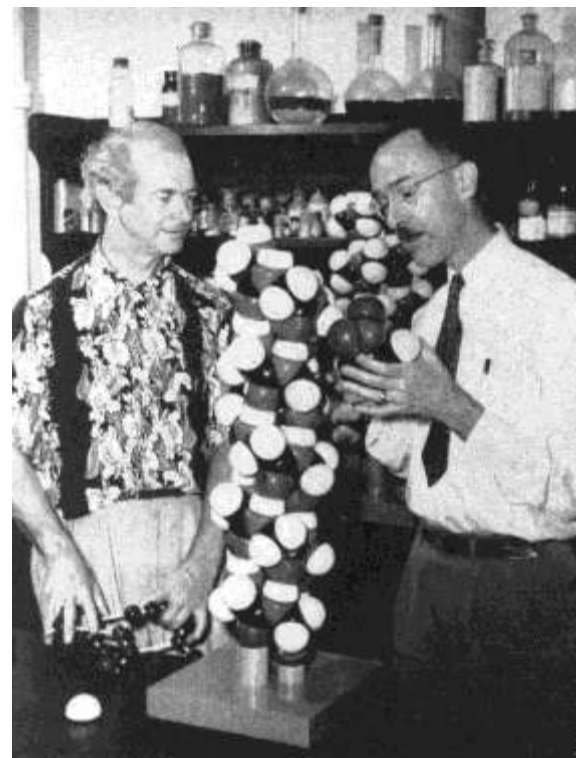
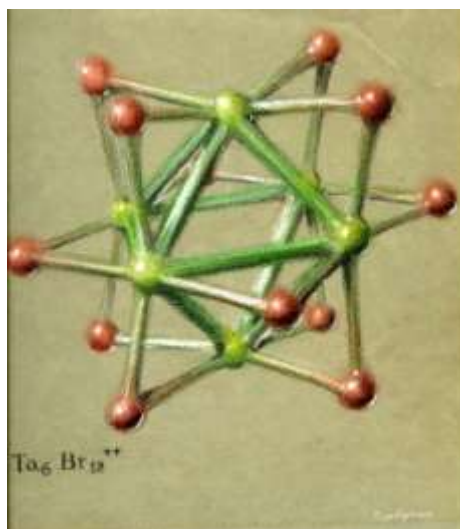
Linus Pauling



The Nobel Prize in Chemistry 1954



Linus Carl Pauling



Linus Pauling & Roger Hayward

The Nobel Prize in Chemistry 1954 was awarded to Linus Pauling *"for his research into the nature of the chemical bond and its application to the elucidation of the structure of complex substances"*.



The Nobel Prize in Chemistry 1962

Max F. Perutz, John C. Kendrew



The Nobel Prize in Chemistry 1962



Max Ferdinand
Perutz

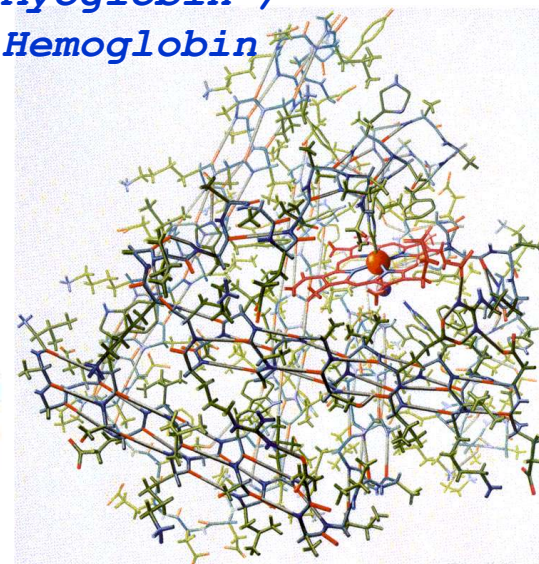


John Cowdery
Kendrew

The Nobel Prize in Chemistry 1962 was awarded jointly to Max Ferdinand Perutz and John Cowdery Kendrew *"for their studies of the structures of globular proteins"*



Myoglobin /
Hemoglobin





The Nobel Prize in Physiology or Medicine 1962

Francis Crick, James Watson, Maurice Wilkins



The Nobel Prize in Physiology or Medicine 1962



Francis Harry
Compton Crick

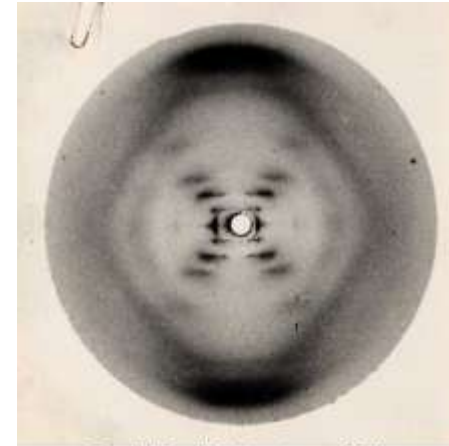


James Dewey Watson

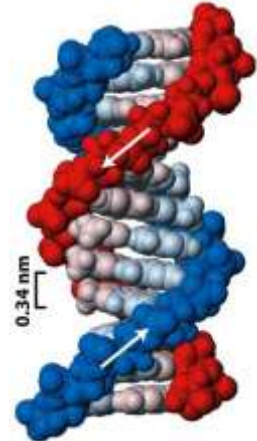


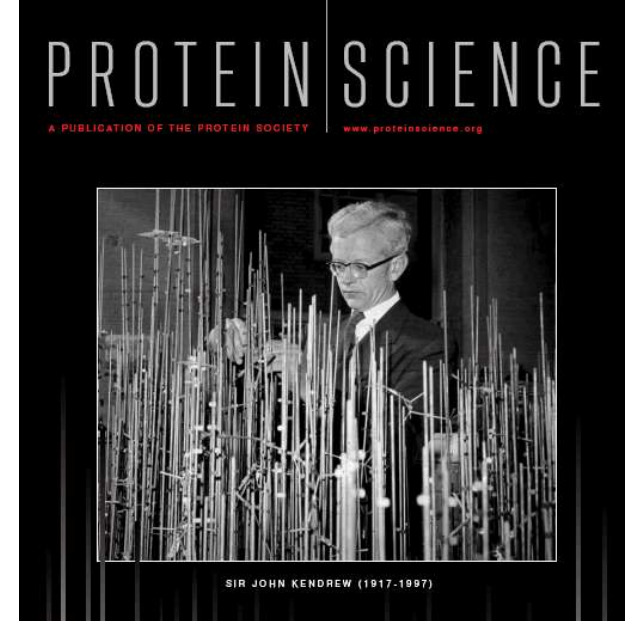
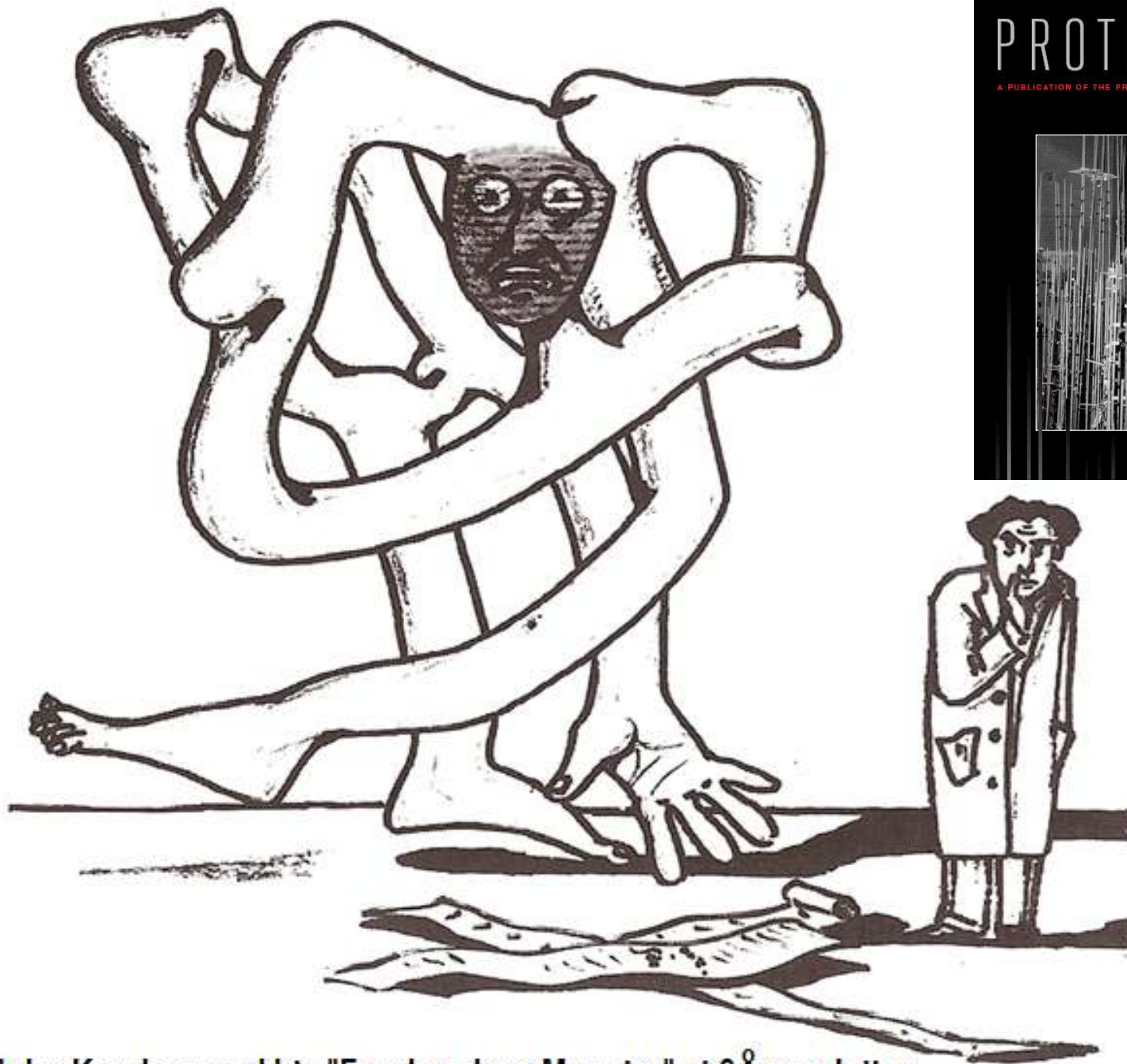
Maurice Hugh
Frederick Wilkins

The Nobel Prize in Physiology or Medicine 1962 was awarded jointly to Francis Harry Compton Crick, James Dewey Watson and Maurice Hugh Frederick Wilkins *"for their discoveries concerning the molecular structure of nucleic acids and its significance for information transfer in living material"*.



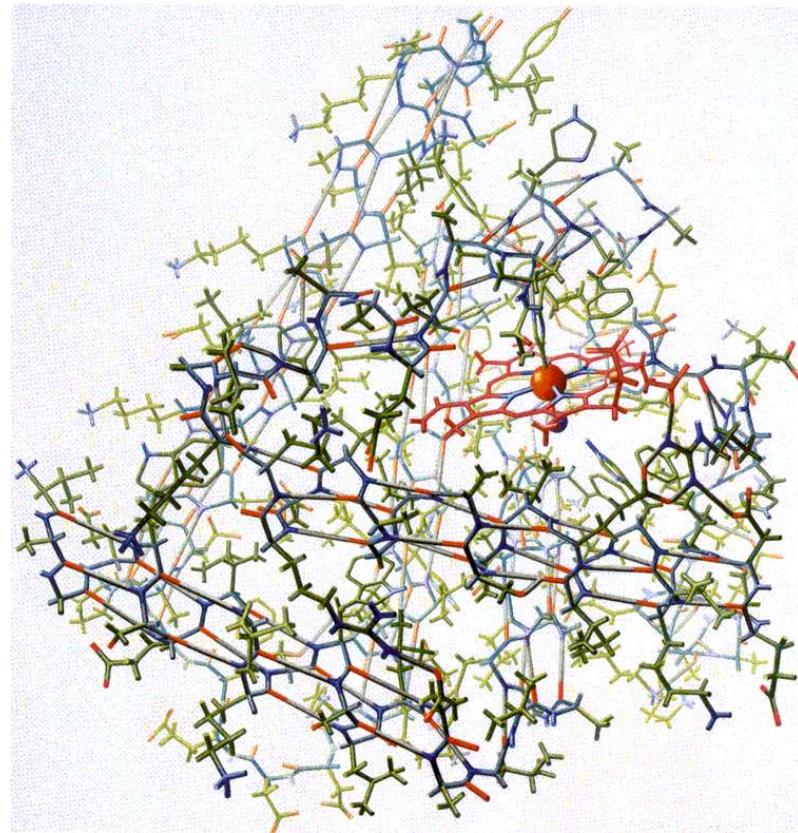
Rosalind Franklin's X-ray image of DNA





John Kendrew and his "Frankendrew Monster" at 6Å resolution

DICKERSON AND GEIS
THE STRUCTURE AND ACTION OF
PROTEINS



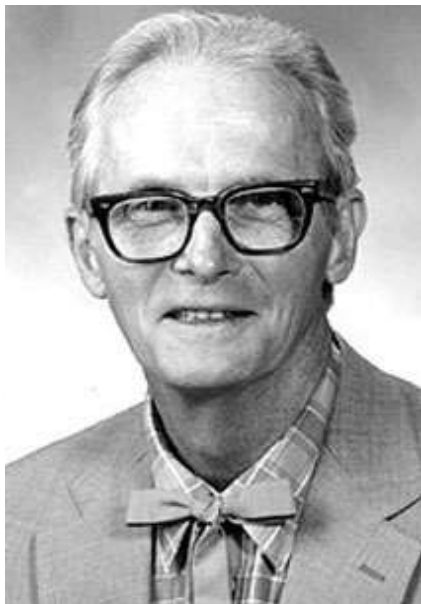


The Nobel Prize in Chemistry 1976

William Lipscomb



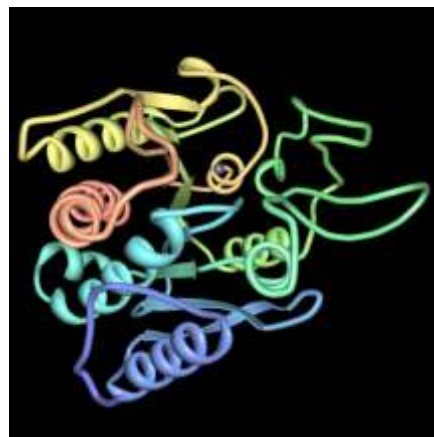
The Nobel Prize in Chemistry 1976



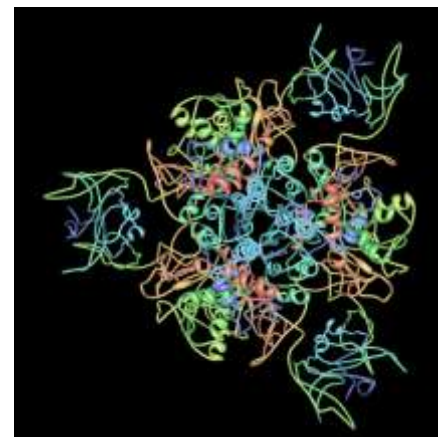
William N. Lipscomb



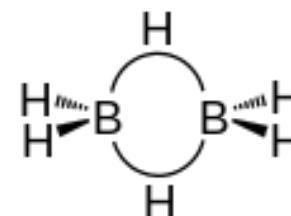
Lipscombite



Carboxypeptidase A



*Aspartate
carbamoyl
transferase*



*Bonding diagram of **diborane** (B_2H_6) showing with curved lines a pair of 3-center 2-electron bonds.*

The Nobel Prize in Chemistry 1976 was awarded to William Lipscomb *"for his studies on the structure of boranes illuminating problems of chemical bonding"*.



The Nobel Prize in Chemistry 1985

Herbert A. Hauptman, Jerome Karle



The Nobel Prize in Chemistry 1985



Herbert A. Hauptman



Jerome Karle

$$F_{hkl} = \sum_{h'k'l'} F_{h'k'l'} F_{h-h', k-k', l-l'}$$

Sayre equation

$$S_h \approx S_{h'} S_{h-h'}$$

Direct methods are the preferred method for phasing crystals of small molecules having up to 1000 atoms in the asymmetric unit. Hauptman and Karle developed a practical method to employ the Sayre equation to exploit statistical correlations between the phases of different Fourier components that result from the fact that the scattering density must be a positive real number.

The Nobel Prize in Chemistry 1985 was awarded jointly to Herbert A. Hauptman and Jerome Karle *"for their outstanding achievements in the development of direct methods for the determination of crystal structures"*



The Nobel Prize in Chemistry 1988

Johann Deisenhofer, Robert Huber, Hartmut Michel



The Nobel Prize in Chemistry 1988



Johann Deisenhofer

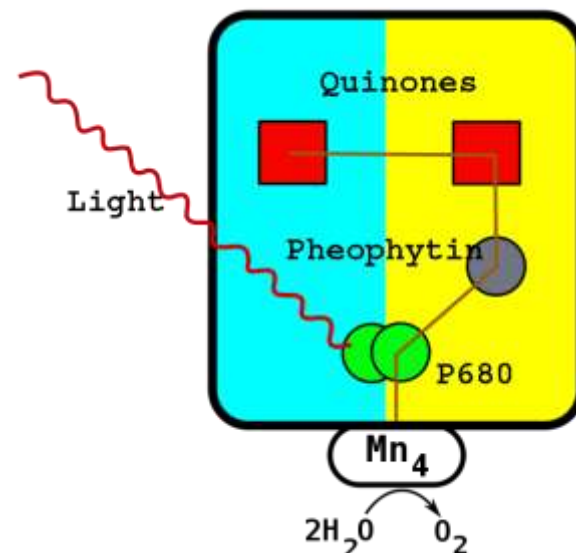
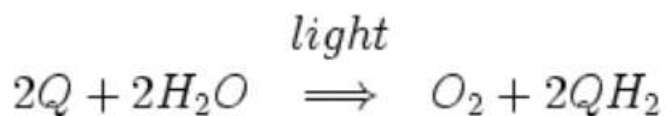


Robert Huber

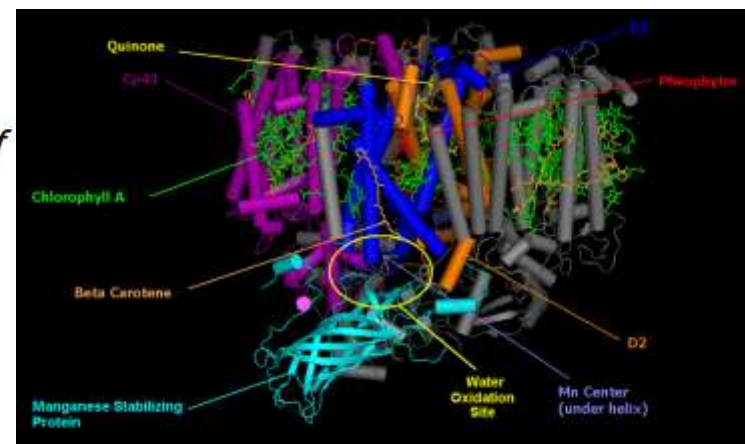


Hartmut Michel

The Nobel Prize in Chemistry 1988 was awarded jointly to Johann Deisenhofer, Robert Huber and Hartmut Michel *"for the determination of the three-dimensional structure of a photosynthetic reaction centre"*.



Photoreaction Center





The Nobel Prize in Chemistry 2003

Peter Agre, Roderick MacKinnon



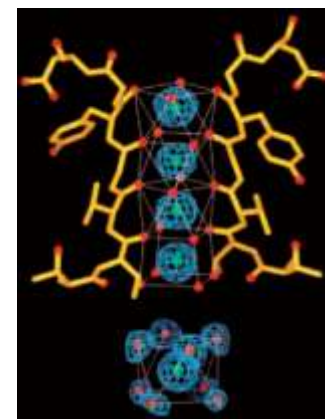
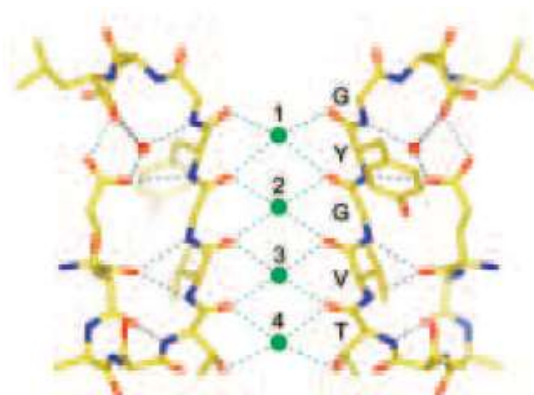
The Nobel Prize in Chemistry 2003



Peter Agre



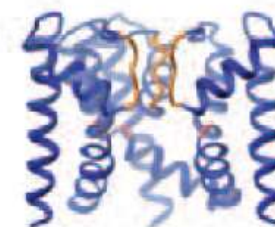
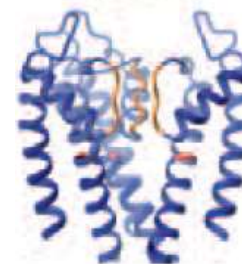
Roderick MacKinnon



Potassium Ion
Channels

Closed

Opened



The Nobel Prize in Chemistry 2003 was awarded *"for discoveries concerning channels in cell membranes"* jointly with one half to Peter Agre *"for the discovery of water channels"* and with one half to Roderick MacKinnon *"for structural and mechanistic studies of ion channels"*.



The Nobel Prize in Chemistry 2009

Venkatraman Ramakrishnan, Thomas A. Steitz, Ada E. Yonath



The Nobel Prize in Chemistry 2009



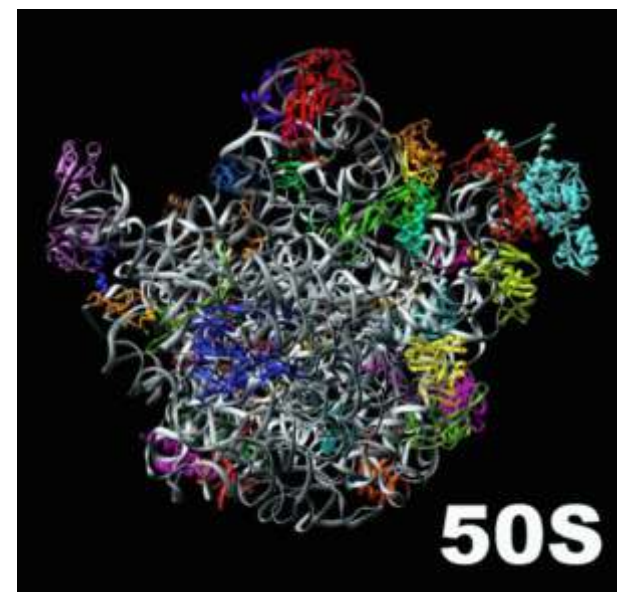
Photo: U. Montan
Venkatraman
Ramakrishnan



Photo: U. Montan
Thomas A. Steitz



Photo: U. Montan
Ada E. Yonath



The Nobel Prize in Chemistry 2009 was awarded jointly to Venkatraman Ramakrishnan, Thomas A. Steitz and Ada E. Yonath *"for studies of the structure and function of the ribosome"*.

Ribosome



The Nobel Prize in Chemistry 2011

Dan Shechtman



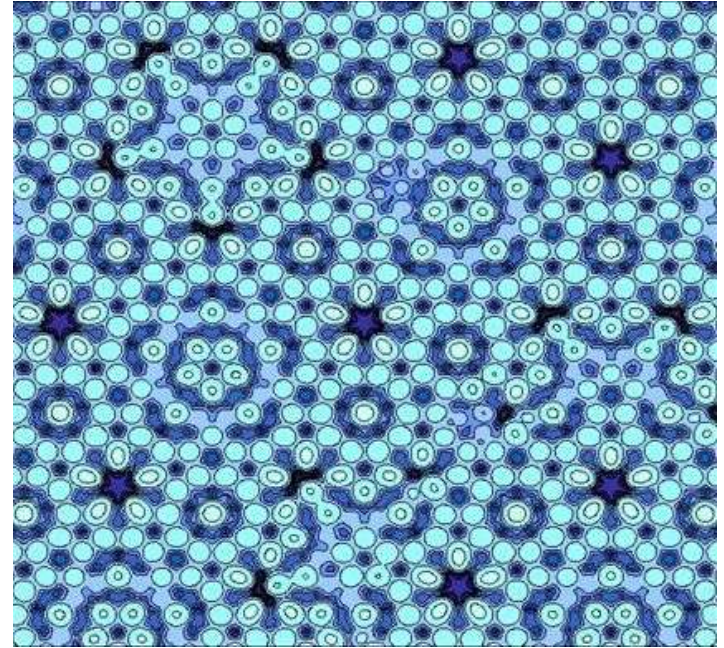
The Nobel Prize in Chemistry 2011



Dan Shechtman

Linus Pauling –
*"There is no such thing as
quasicrystals, only quasi-
scientists."*

*Quasicrystals have low
thermal and electrical
conductivity, while
possessing high structural
stability – ideal for non-
stick insulation for
electrical wires and
cooking equipment.*



*Quasicrystals – ordered
crystalline materials lacking
repeating structures, such as
this Al-Pd-Mn alloy.*

The Nobel Prize in Chemistry 2011 was awarded to Dan Shechtman *"for the discovery of quasicrystals"*.



The Nobel Prize in Chemistry 2013

Martin Karplus, Michael Levitt, Arieh Warshel



The Nobel Prize in Chemistry 2013



© Harvard University

Martin Karplus



Photo: © S. Fisch

Michael Levitt



Photo: Wikimedia Commons

Arieh Warshel

The Nobel Prize in Chemistry 2013 was awarded jointly to Martin Karplus, Michael Levitt and Arieh Warshel *"for the development of multiscale models for complex chemical systems"*.



The Nobel Prize in Chemistry 1964

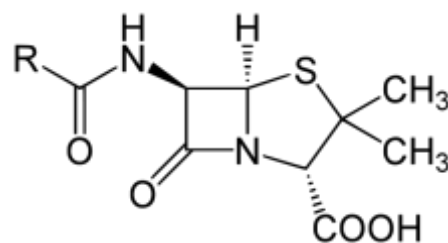
Dorothy Crowfoot Hodgkin



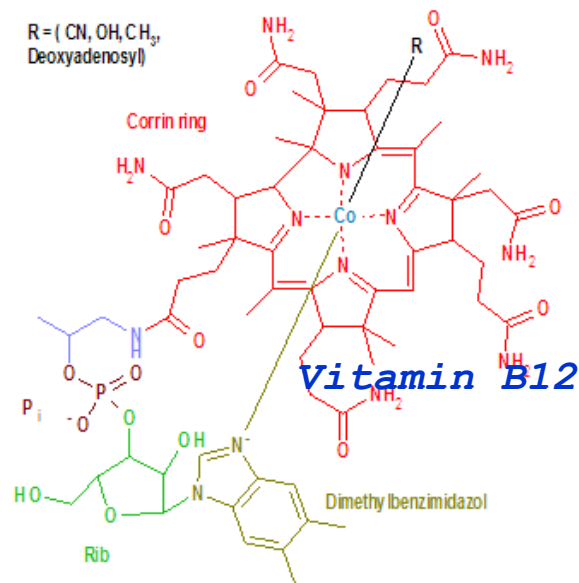
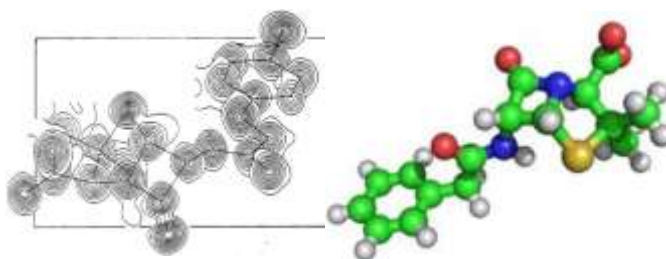
The Nobel Prize in Chemistry 1964



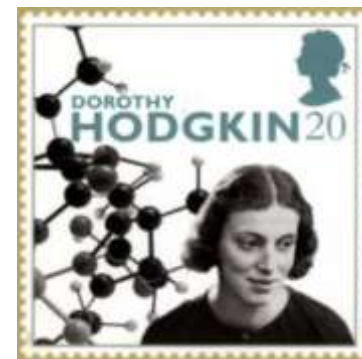
Dorothy Crowfoot Hodgkin



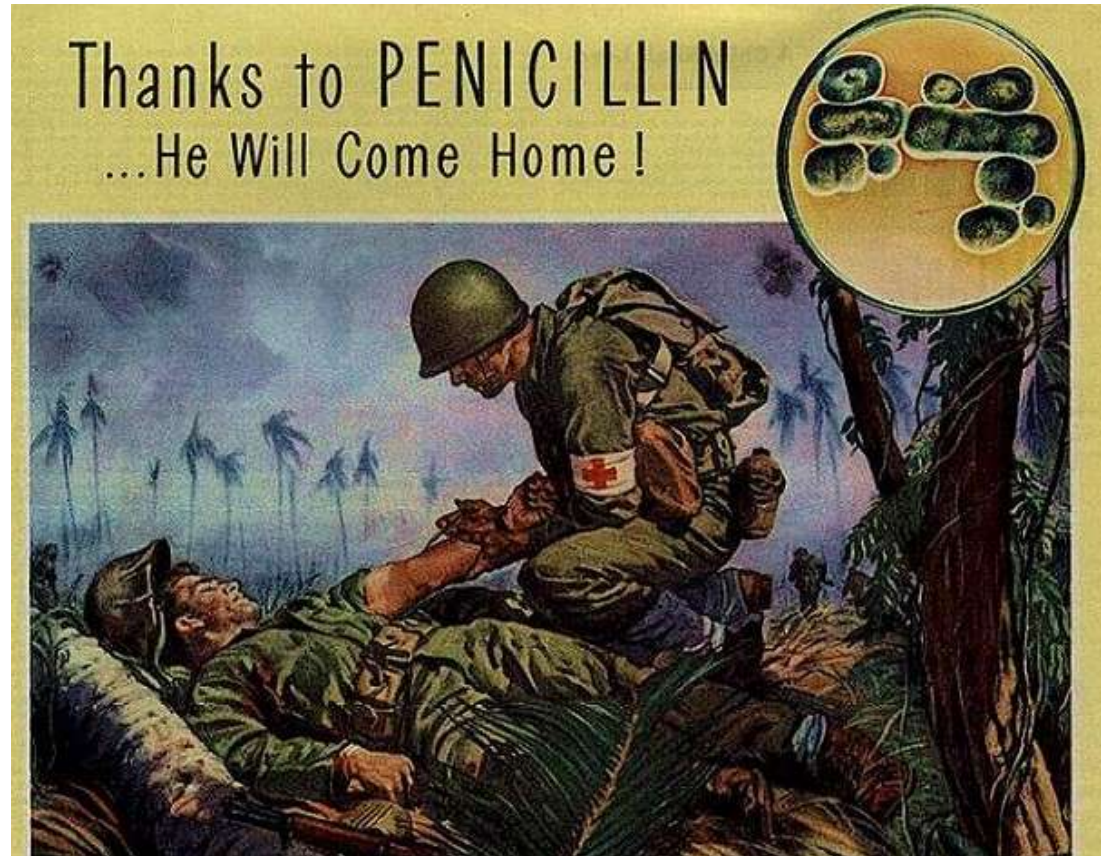
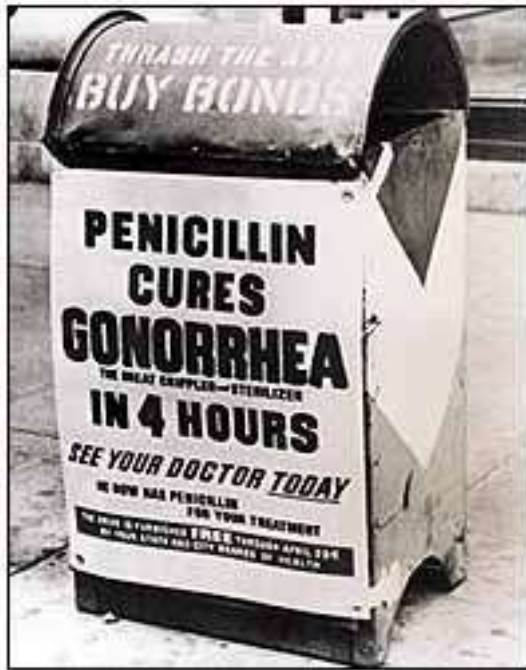
Penicillin



The Nobel Prize in Chemistry 1964 was awarded to Dorothy Crowfoot Hodgkin *"for her determinations by X-ray techniques of the structures of important biochemical substances"*.



The Good Old Days...



**modern
headlines:**

The ticking time bomb...

The serious threat...

escalating antibiotic resistance...

untreatable bacterial infections...

- Alexander Fleming (Scottish scientist) fought in World War I. His battlefield experience showed him how dangerous bacteria could be to human life. After the war he spent most of his time at **St. Mary's Hospital in London** doing research, focusing on finding a chemical that would stop bacterial infection.
- He discovered lysozyme, an enzyme occurring in many body fluids, such as tears. It had a natural antibacterial effect, but not against the strongest infectious agents (*e.g. Staphylococcus aureus*). Fleming's research focused on finding agents that would kill this particular bacterium.
- In the Fall of 1928, Fleming had prepared *Staphylococcus* on several plates and forgot to put them in the incubator before he went on vacation.
- When he returned from vacation, he discovered that one of the plates was contaminated with molds. This was not unusual; it had happened all the time. He knew that this contamination must have come from the floor below, because at that time there was a group there working with molds. **But what was unusual about this particular strain of molds was that it killed the *Staphylococcus* in the surrounding area.**
- It turned out that the reasons why Fleming was able to observe this effect were: (1) this particular mold strain (later identified to be *Penicillium notatum*) was a **good producer of penicillin**, and (2) the **temperature was unusually cold at that time of year**, which caused the mold and bacteria to grow very slowly (necessary to observe the inhibiting effect).

- For the next 7 years, Fleming and his colleagues tried to produce large quantities of penicillin so that it could be tested in animals and humans, but they not very successful.
- In 1935, **Howard Florey** was appointed professor of pathology at **Oxford** and he hired **Ernst Chain** to work on a cancer-related project. Chain had read the article previously published by Fleming and tried to convince Florey to revisit the penicillin problem (Fleming claimed that it was unstable and that was the reason he could not get enough to do animal or human trials).
- The Florey team began working on penicillin in full force. **Every bucket, container, bottle they could find were used** to culture *Penicillium notatum*. By **May 1940, the team produced enough penicillin to test on infected animals for the first time**. Eight mice were infected with a lethal dose of *Staphylococcus*. One hour later, four of them were injected with penicillin and the other four were left without treatment. All four mice that did not receive penicillin died in less than 24 h. All four mice treated with penicillin were health. Florey proclaimed, **“it looks like a miracle”**.
- **To do test on humans, the amount of pencillin required was 3,000 times greater**. By now, **England was at war** and financial resources were limited. A creative solution would have to be developed in order to produce penicillin in large quantity. **Heatly, a team member, designed a container that resembles a bedpan out of ceramic, suited for their need. 400 stackable containers were made and this allowed them to produce enough penicillin to do test on human for the first time**. Although the first patient treated with penicillin died due to a shortage of supply, further tests showed that penicillin successfully treats human infections as well.

- Florey tried to get British pharmaceutical companies to produce penicillin, but failed (mainly because the war had greater demand for other supplies).
- **In 1941, Florey and Heatly** came to the US to convince American pharmaceutical companies to produce penicillin. **They ended up in Peoria, Illinois. An agriculture research center there had developed excellent techniques of fermentation** (a process needed for penicillin growth). The nutrients used for penicillin growth there was **corn**, which was not commonly grown in Britain. **The penicillin-producing mold loved corn and produced 500 times as much as it had produced before.**
- **The first batches of this new wonder drug** became available in **1943** and were reserved for military use. News of penicillin was suppressed because of the war.
- When it was first released, **it was so valuable that patients' urine was collected and the excreted penicillin purified to be used again.**
- By this time, the US had entered World War II. **The government recruited 21 chemical companies to produce penicillin. From January to May 1943, only 400 million units of penicillin were made; by the time the war ended, US companies were making 650 billion units a month.**
- Penicillin kills bacteria by interfering with aminopetidase, an enzyme responsible for making bacteria cell wall. Human and mammals do not have this enzyme.



The Nobel Prize in Physiology or Medicine 1945

Sir Alexander Fleming, Ernst B. Chain, Sir Howard Florey

The Nobel Prize in Physiology or Medicine 1945



Sir Alexander Fleming



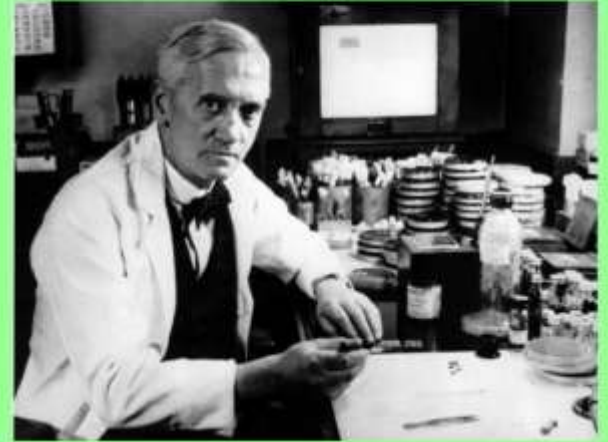
Ernst Boris Chain

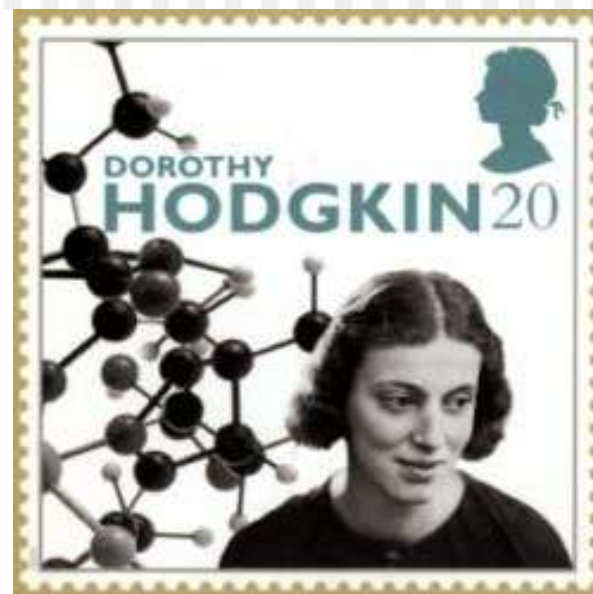
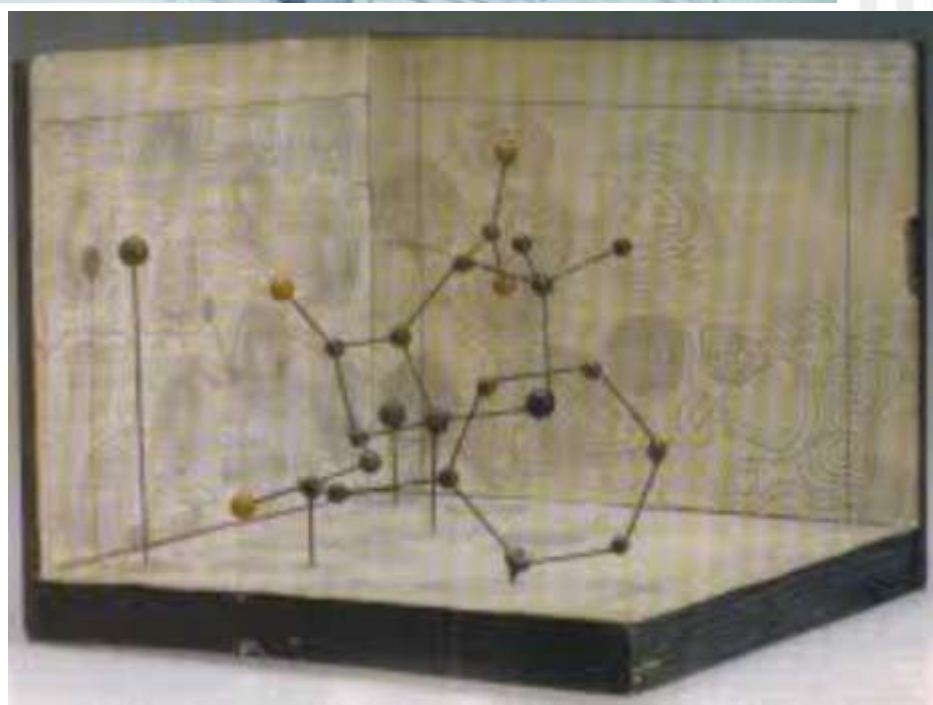
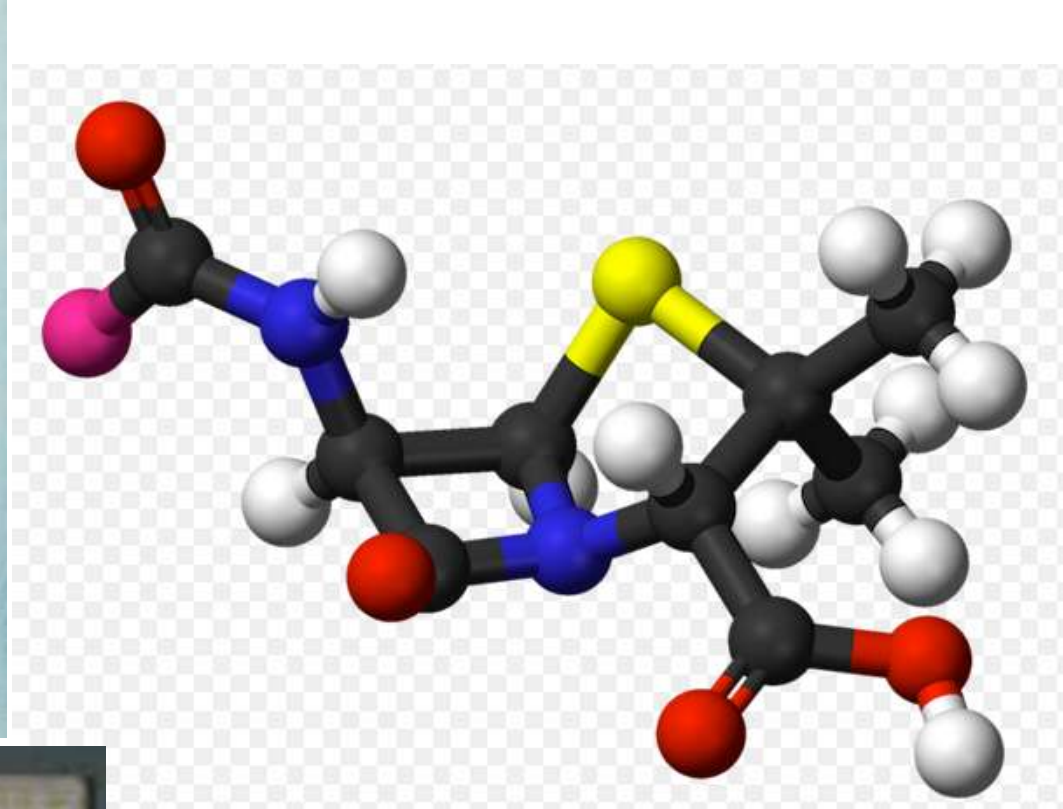
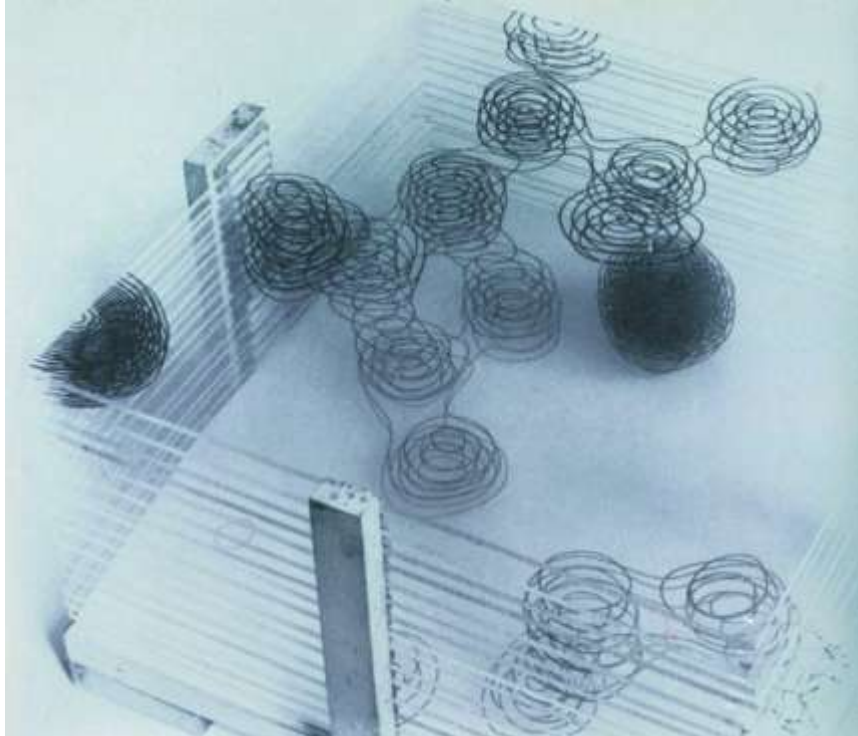


Sir Howard Walter Florey

The Nobel Prize in Physiology or Medicine 1945 was awarded jointly to Sir Alexander Fleming, Ernst Boris Chain and Sir Howard Walter Florey *"for the discovery of penicillin and its curative effect in various infectious diseases"*.

Alexander Fleming and Penicillin: The Accidental Discovery?







Diamond

Pink
Diamond

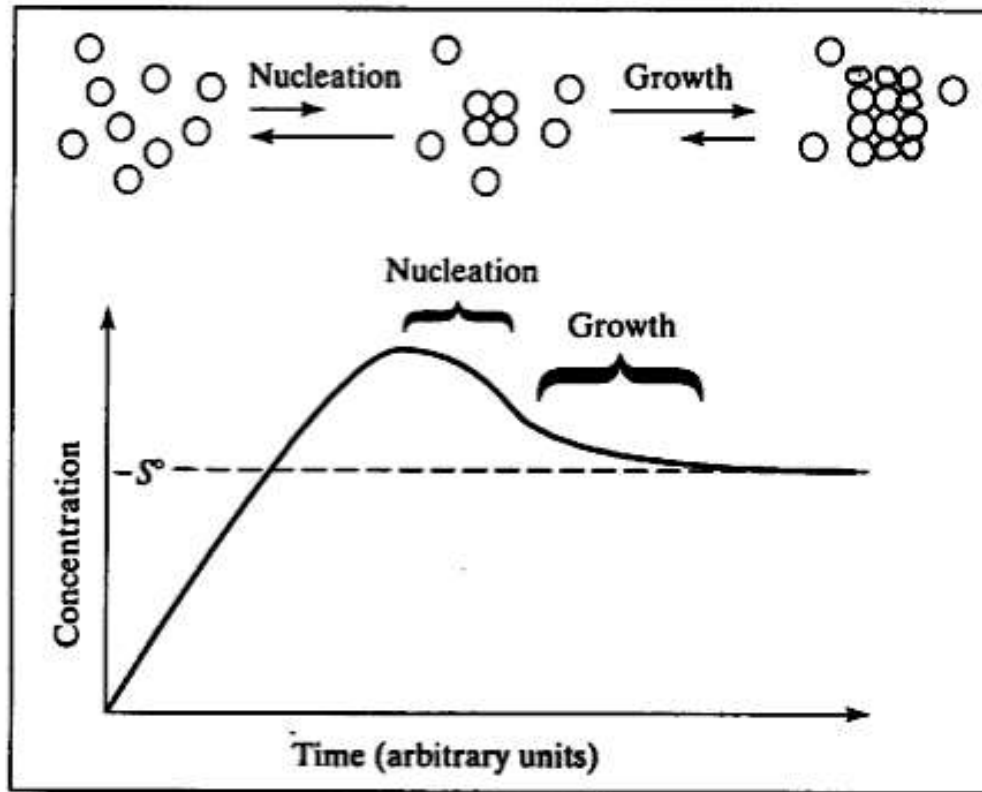


Uncut
Diamond

Crystals

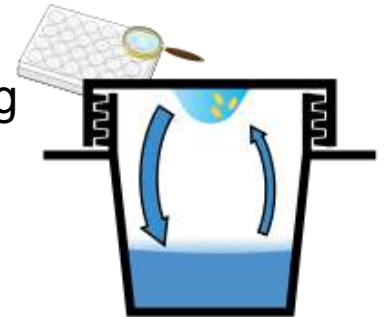


Growing protein crystals.

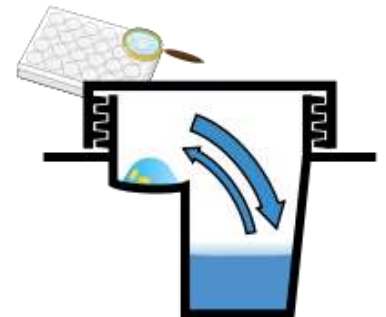


Methods to slowly change solution conditions

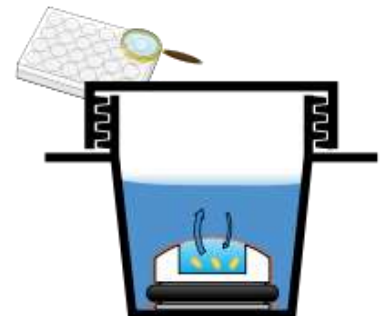
Hanging Drop



Sitting Drop



Micro-dialysis



Variables that influence crystal growth

1. **Nature of macromolecule** – Purity and concentration of macromolecule
2. **Nature and concentration of precipitant**
3. **pH / Temperature / Pressure**
4. **Level of reducing agent or oxidant**
5. **Substrates, coenzymes, and ligands / Metal ions**
6. **Preparation and storage of macromolecule / Proteolysis and fragmentation**
7. **Age of macromolecule / Degree of denaturation**
8. **Vibration and sound**
9. **Volume of crystallization sample**
10. **Seeding**
11. **Amorphous precipitate**
12. **Buffers**
13. **Cleanliness**
14. **Organism or species from which the macromolecule was isolated**
15. **Gravity, gradients and convection**

Common Compounds used in Crystallization

Ammonium sulfate / or sodium

Sodium or ammonium citrate

Sodium or ammonium acetate

Magnesium sulfate

Cetyltrimethyl ammonium salts

Polyethylene glycol 400, 1000, 2000, 4000, 6000, 8000, 15,000 M

Methods for protein crystallization

Batch crystallization (simply dump reagents together)

Liquid-liquid diffusion in a capillary tube

Vapor diffusion-the most successful method (**hanging drop**, **sitting drop**), typically using a Limbro plate. Equilibration occurs between the liquid and vapor phase.

Dialysis

Hanging Drop Method - Crystal Screening

The Experimental Setup

In order to obtain a crystal, the protein molecules must assemble into a periodic lattice. One starts with a solution of the protein with a fairly high concentration (2-50 mg/ml) and adds reagents that reduce the solubility close to spontaneous precipitation. By slow further concentration, and under conditions suitable for the formation of a few nucleation sites, small crystals **may** start to grow. Often very many conditions have to be tried to succeed. This is usually done by initial screening, followed by a systematic optimization of conditions. Crystals should be a few tenth of a mm in each direction to be useful for diffraction experiments.

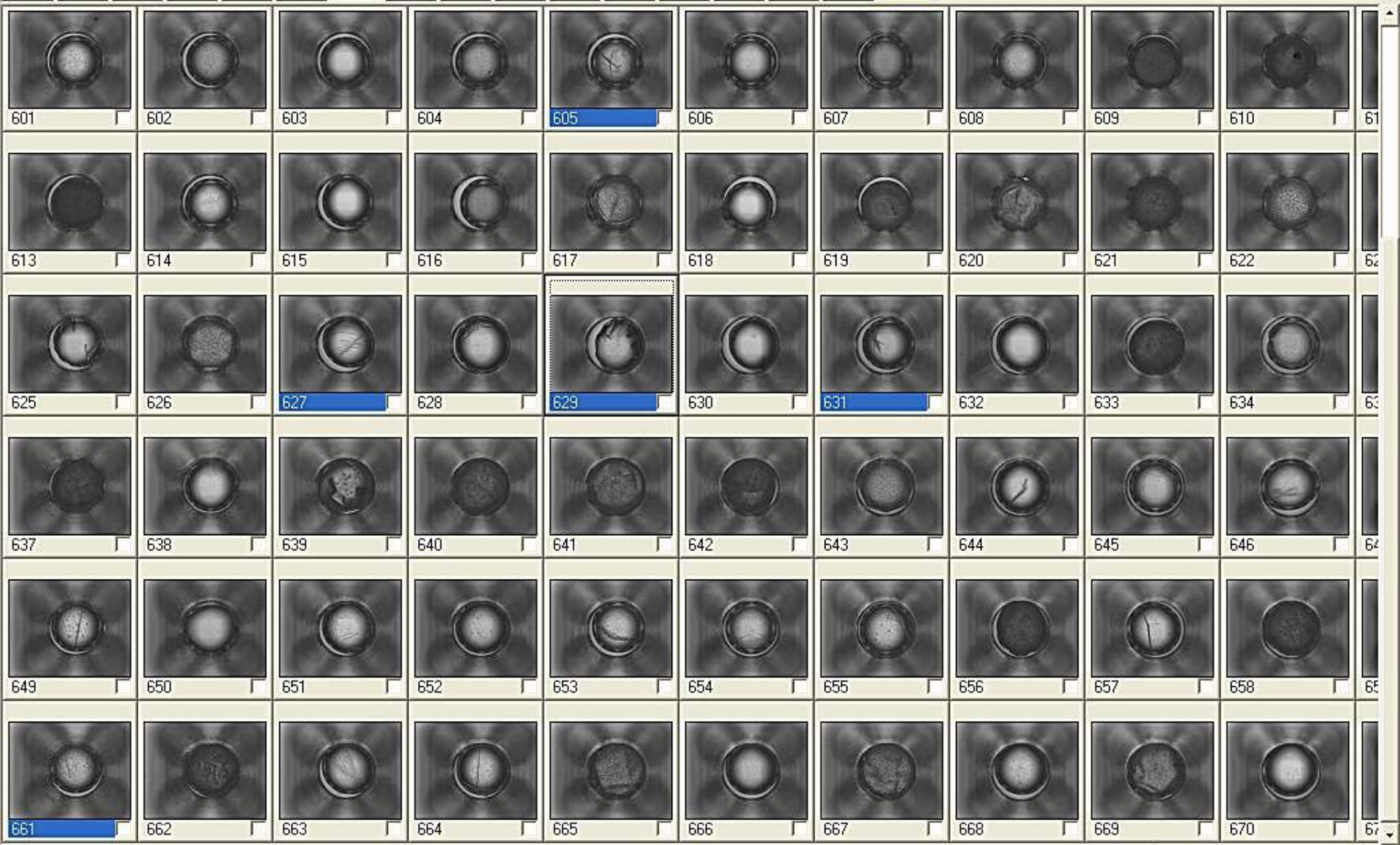


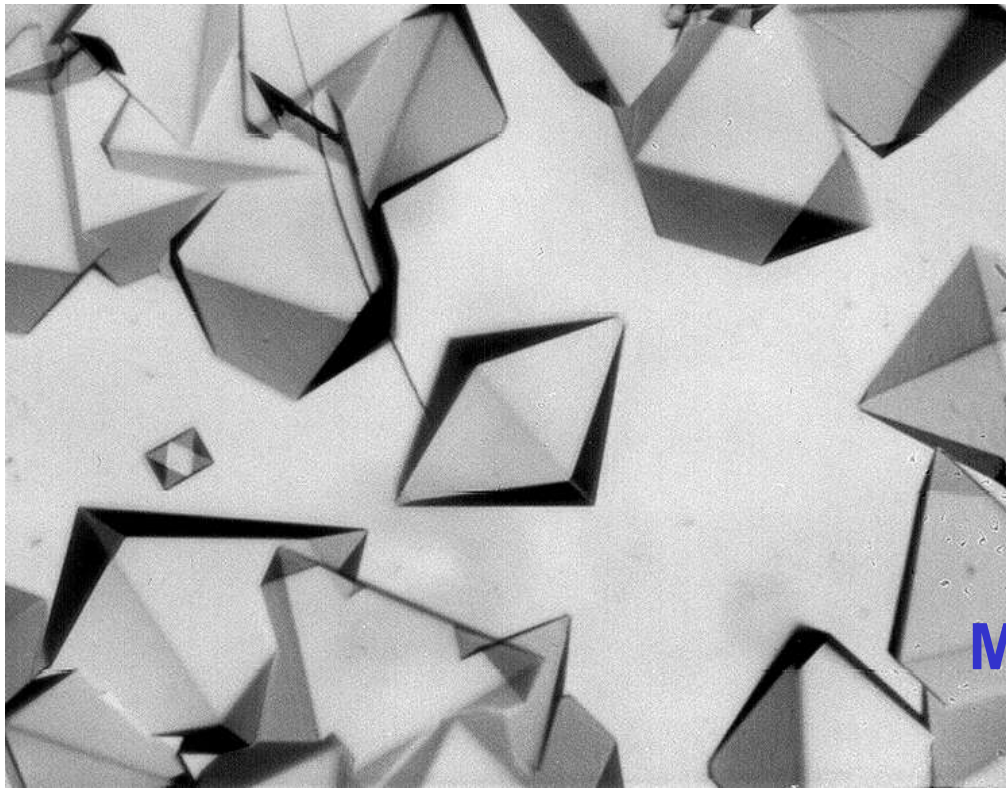
Hampton Crystal Screen Solutions



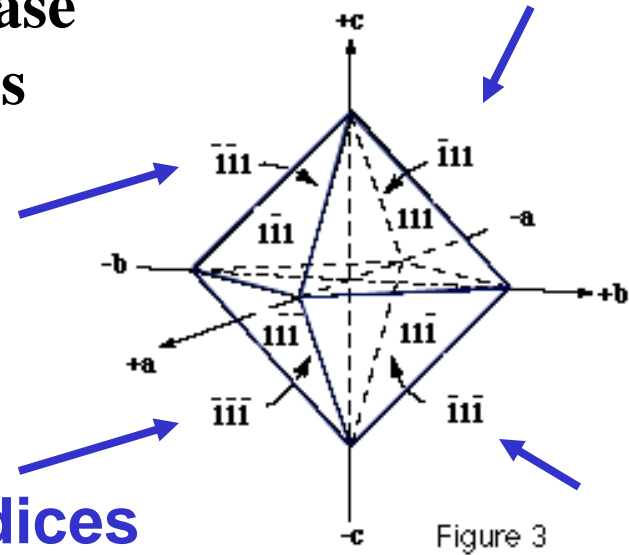
Tube #	SALT	BUFFER	Precipitant	Miniscreen	Tube #
1	0.02M Calcium Chloride	0.1M Na Acetate pH 4.6	30% w/v 2-methyl-2,4-pentanediol	Y	1
2	None	None	0.4M K,Na Tartrate tetrahydrate		2
3	None	None	0.4M Ammonium dihydrogen phosphate		3
4	None	0.1M Tris-HCl pH 8.5	2.0M Ammonium Sulfate	Y	4
5	0.2M tri-sodium citrate	0.1M Na HEPES pH 7.5	30% w/v 2-methyl-2,4-pentanediol		5
6	0.2M Magnesium chloride	0.1M Tris-HCl pH 8.5	30% w/v PEG 4000		6
7	None	0.1M Na Cacodylate pH 6.5	1.4M Sodium acetate trihydrate		7
8	0.2M tri-sodium citrate	0.1M Na Cacodylate pH 6.5	30% v/v 2-propanol		8
9	0.2M Ammonium acetate	0.1M Na Citrate pH 5.6	30% w/v PEG 4000	Y	9
10	0.2M Ammonium acetate	0.1M Na Acetate pH 4.6	30% w/v PEG 4000	Y	10

1 plate 2 plate 3 plate 4 plate 5 plate 6 plate 7 plate 8 plate 9 plate 10 plate 11 plate 12 plate 13 plate 14 plate 15 plate 16



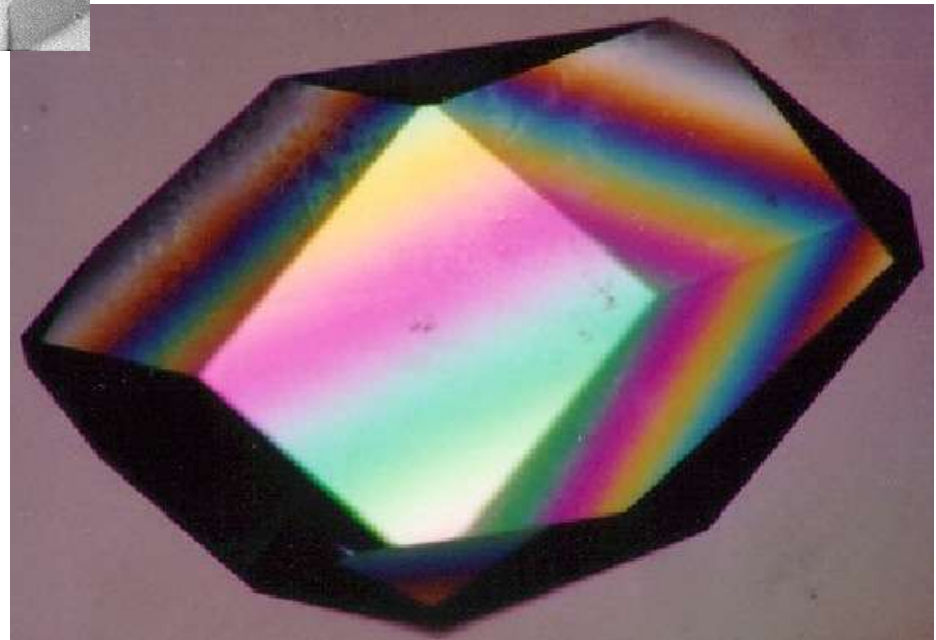


Glucose Isomerase Crystals



Miller Indices

Lysozyme Crystals



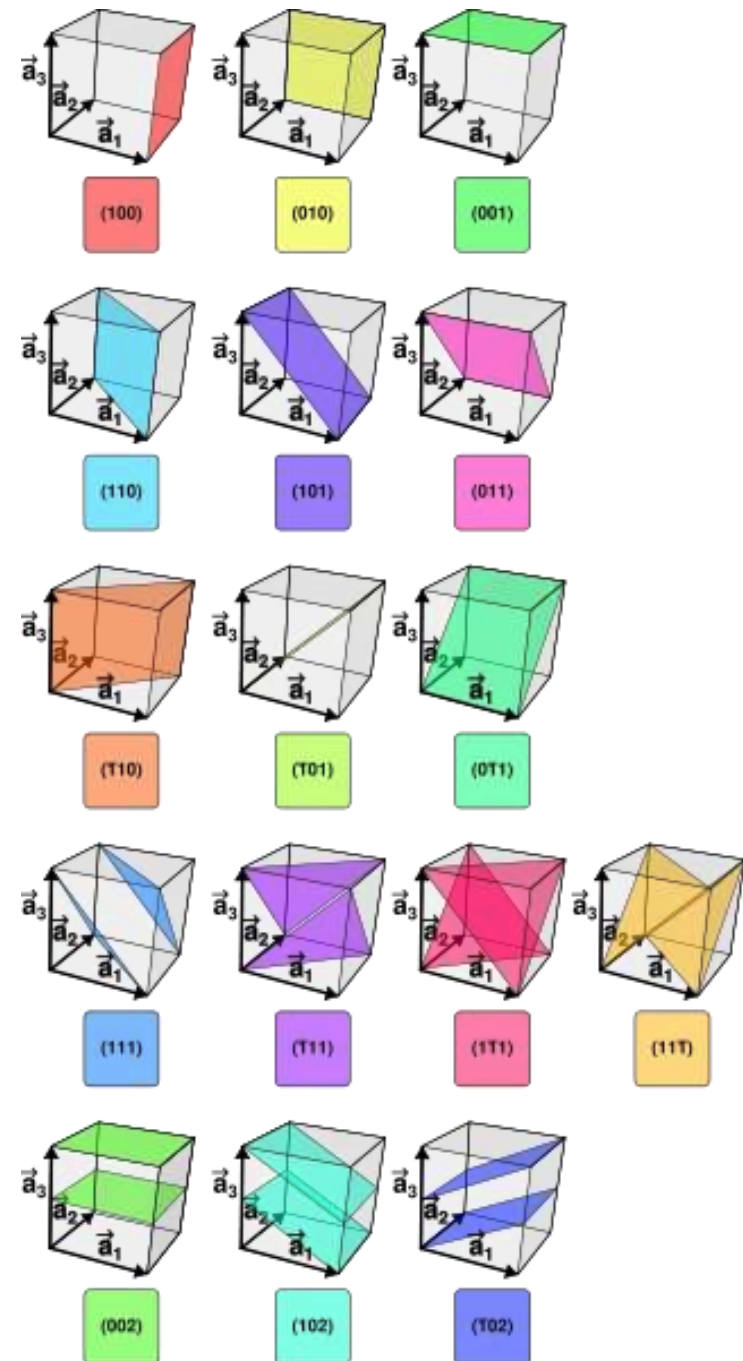
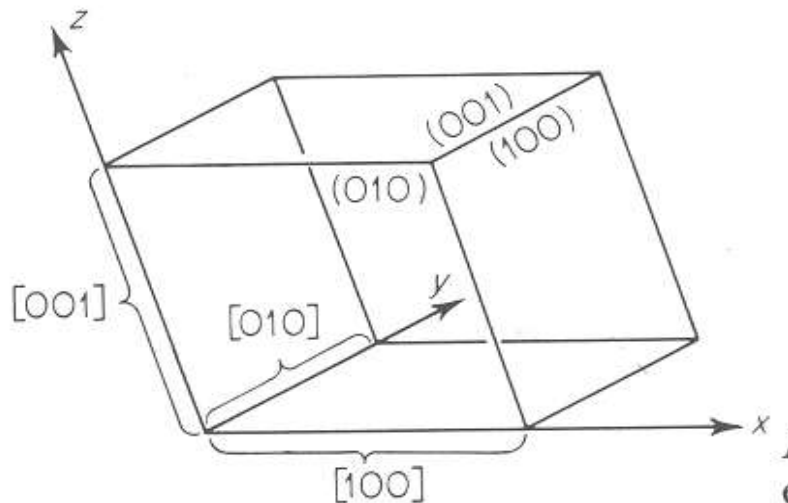
Bragg Planes

X-ray scattering can be understood as if the x-rays are reflected from planes in the crystal.

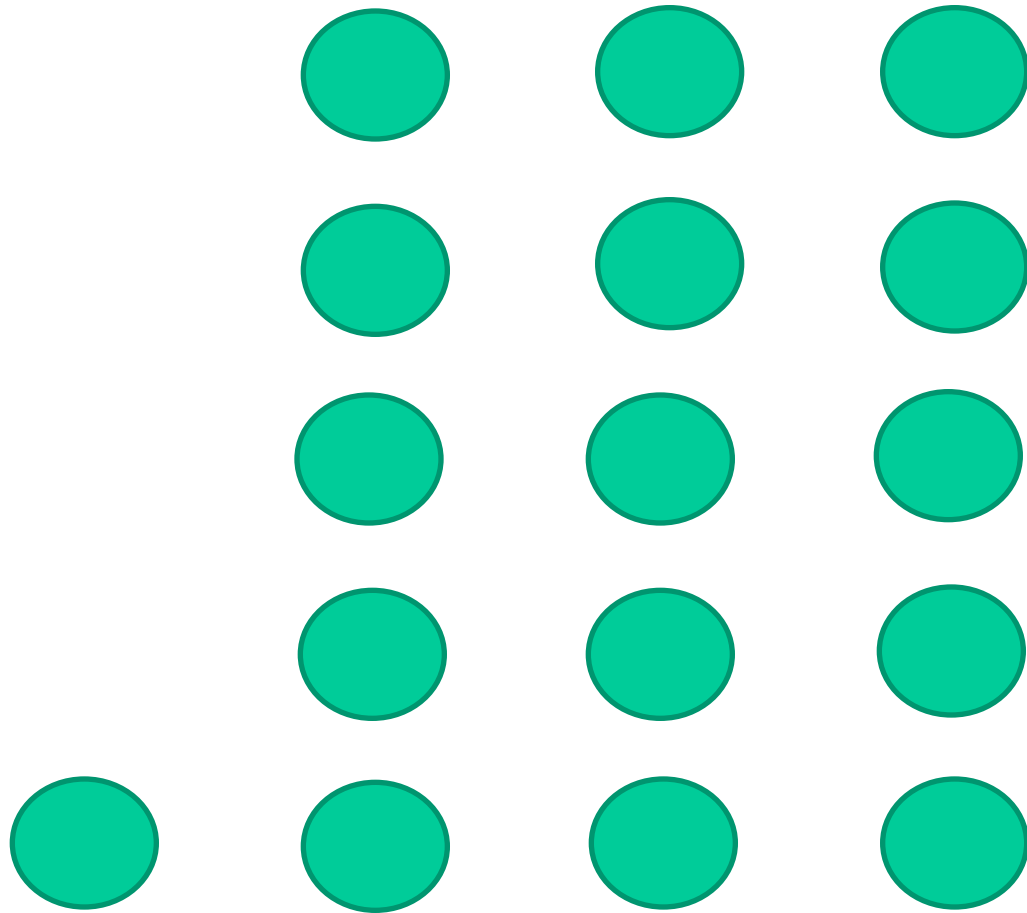
h, k, l are integers, called “Miller indices”

Miller indices are used to define the direction of planes within a crystal. A plane with Miller indices h, k, l intersects the unit cell edges a, b, c at points $a/h, b/k$ and c/l .

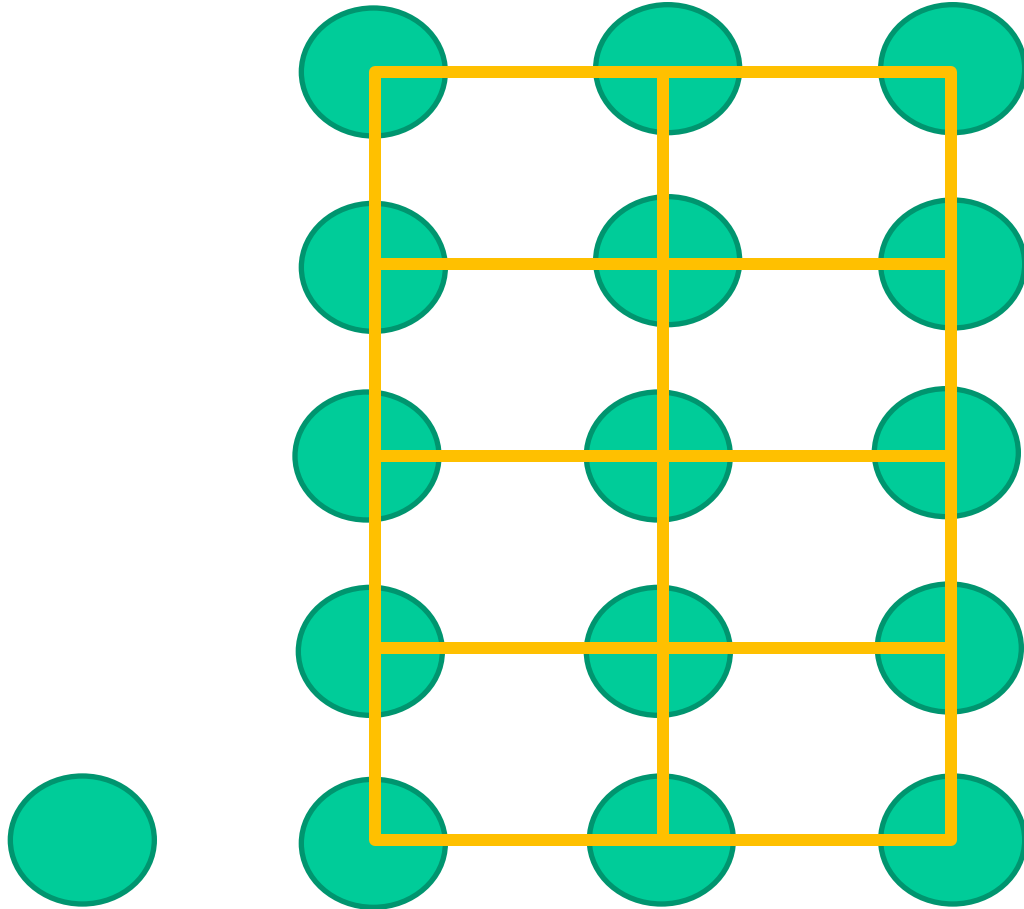
(3 integers define the direction of a plane).



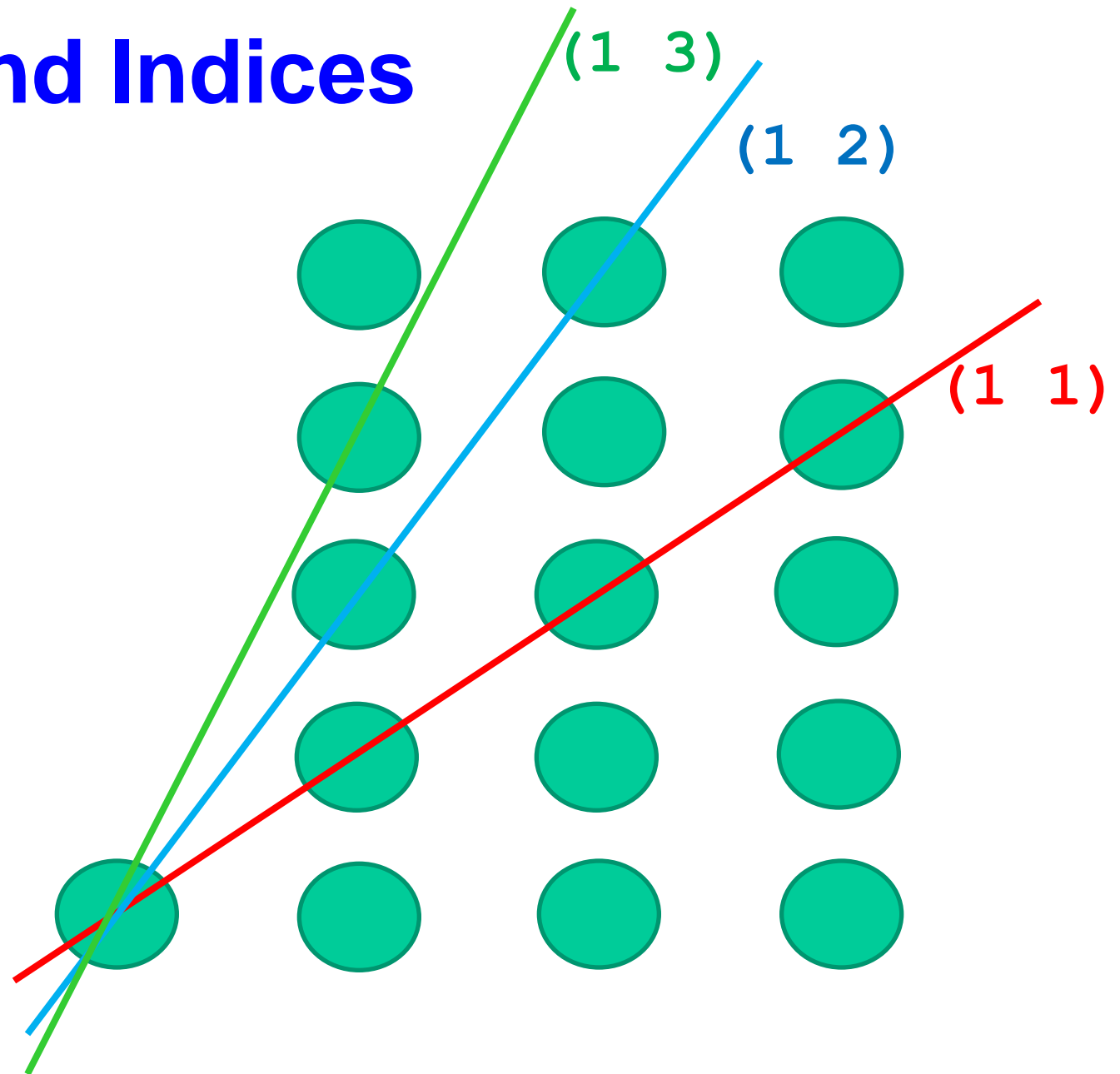
Atoms in space



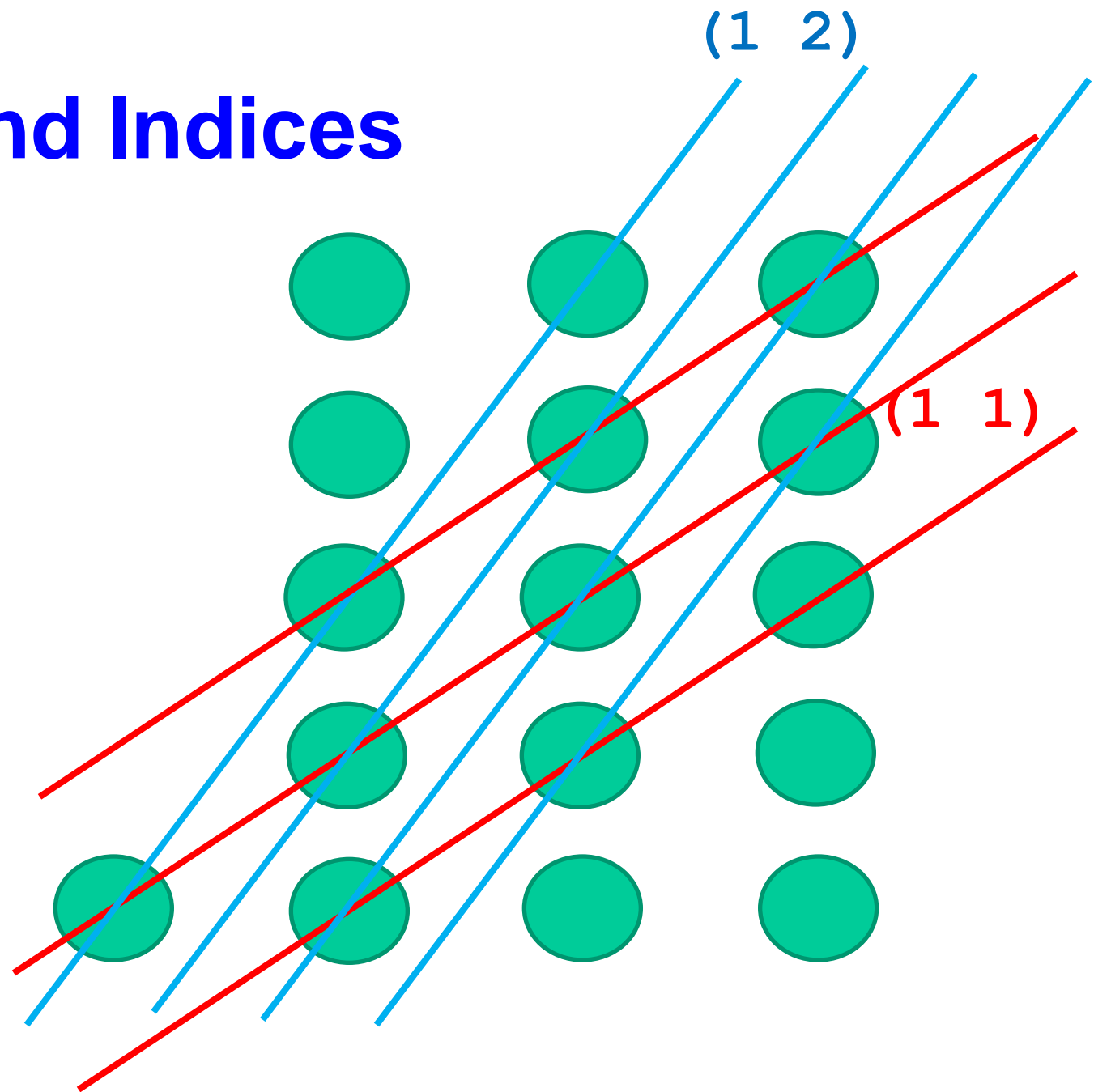
Atoms in space / Unit Cell



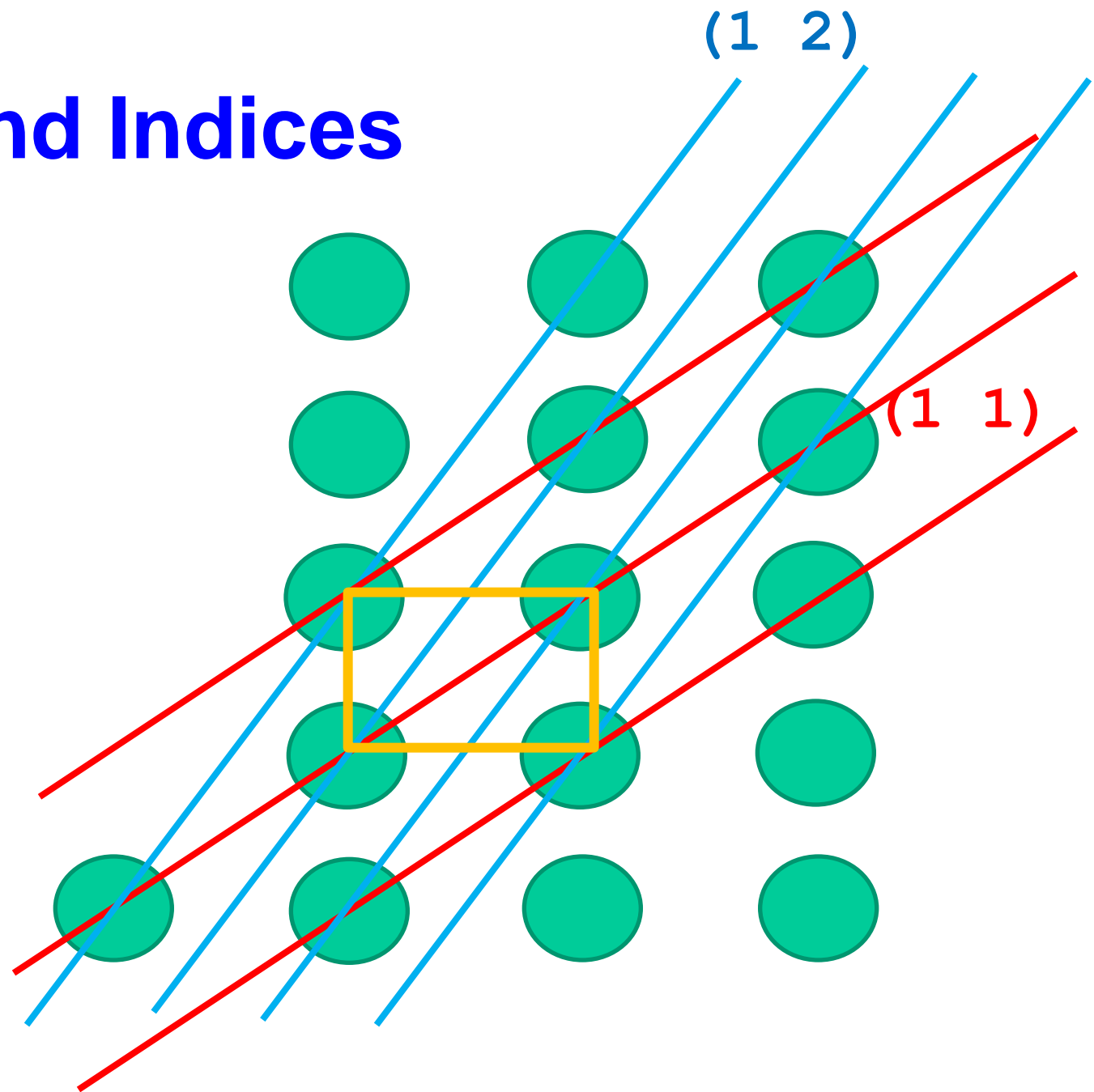
Planes and Indices



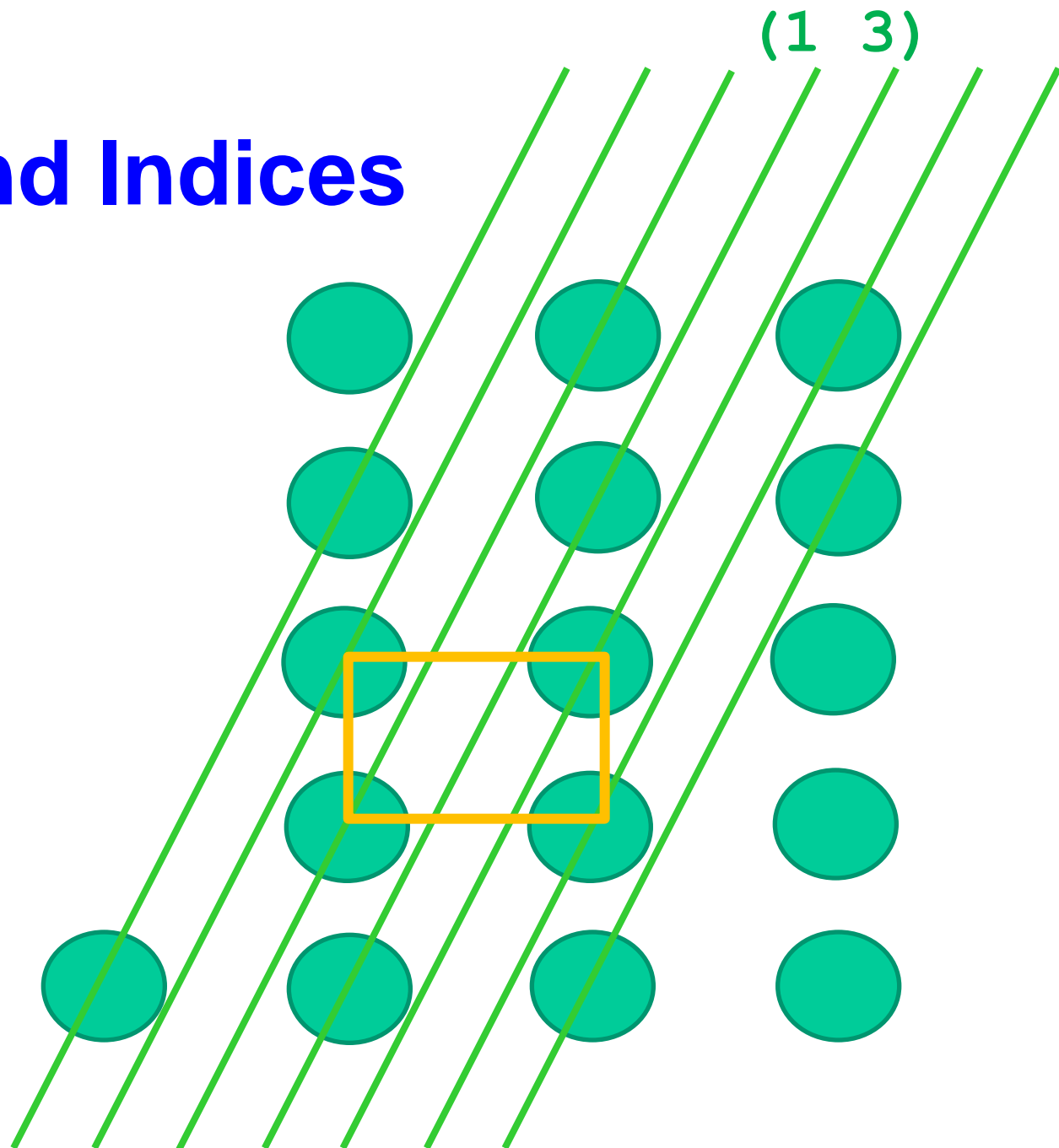
Planes and Indices



Planes and Indices

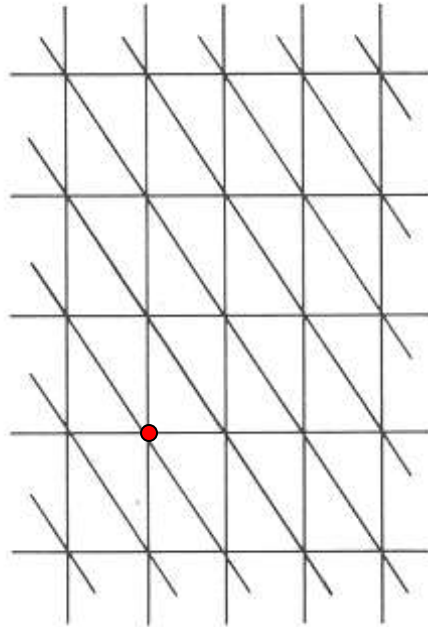


Planes and Indices



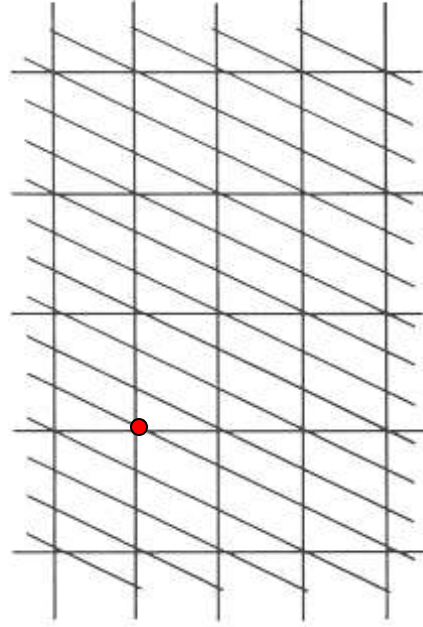
Name that Bragg “plane”

1 1 0



(a)

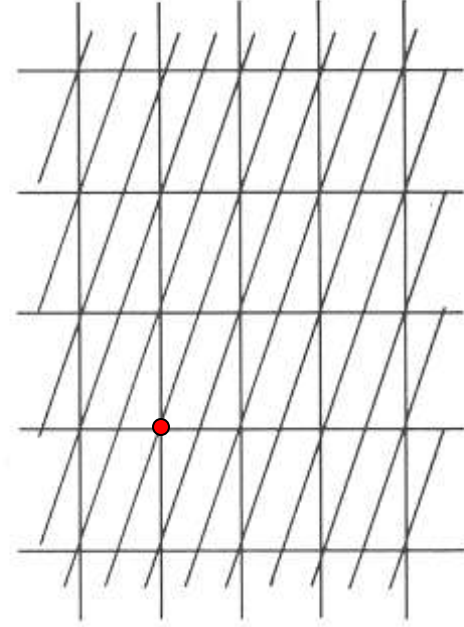
1 3 0



(b)

2 -1 0

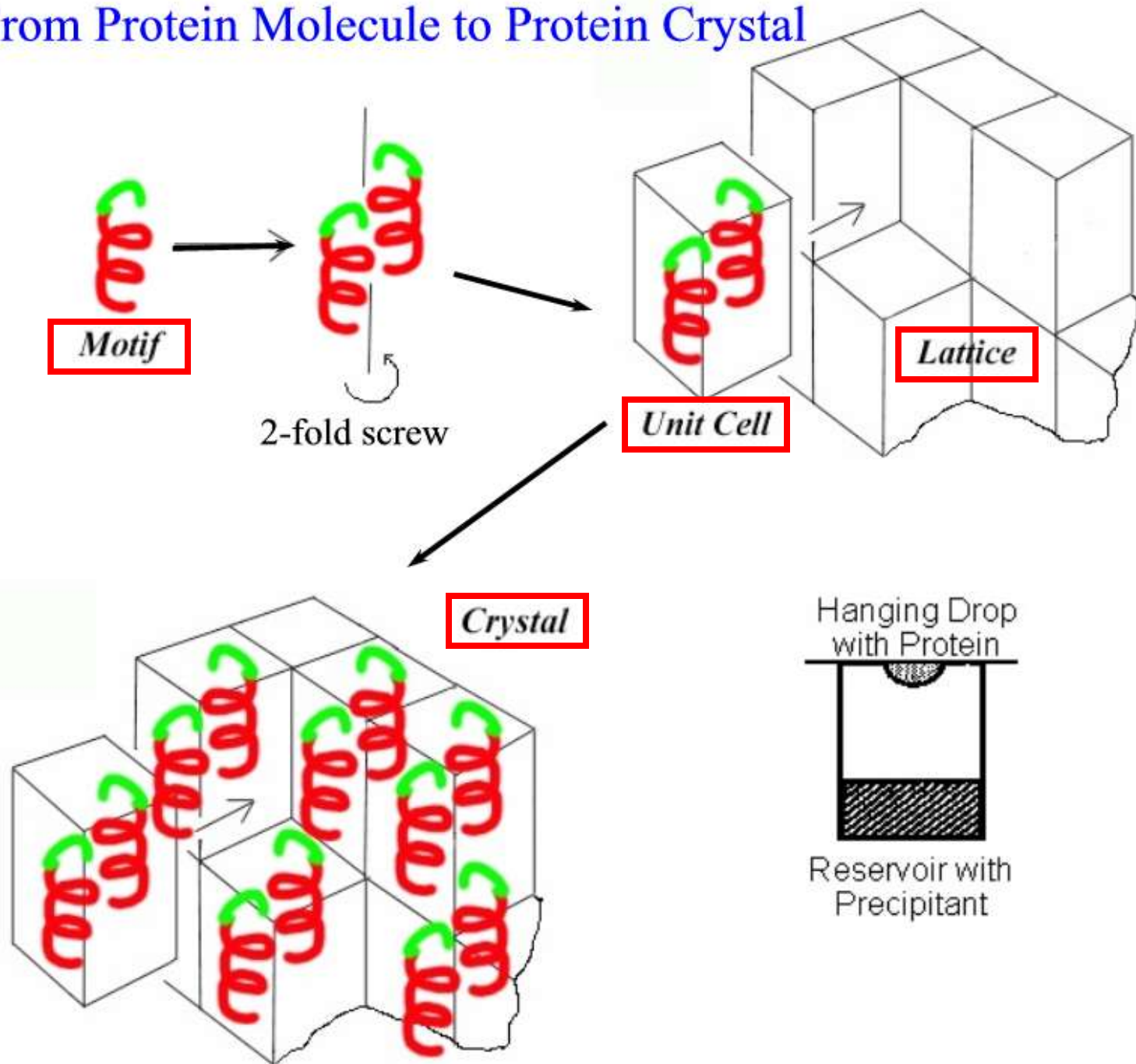
(or -2 1 0)



(c)

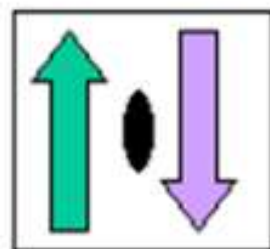
Figure 2.5. Three families of lattice “planes” in a two-dimensional lattice.

From Protein Molecule to Protein Crystal

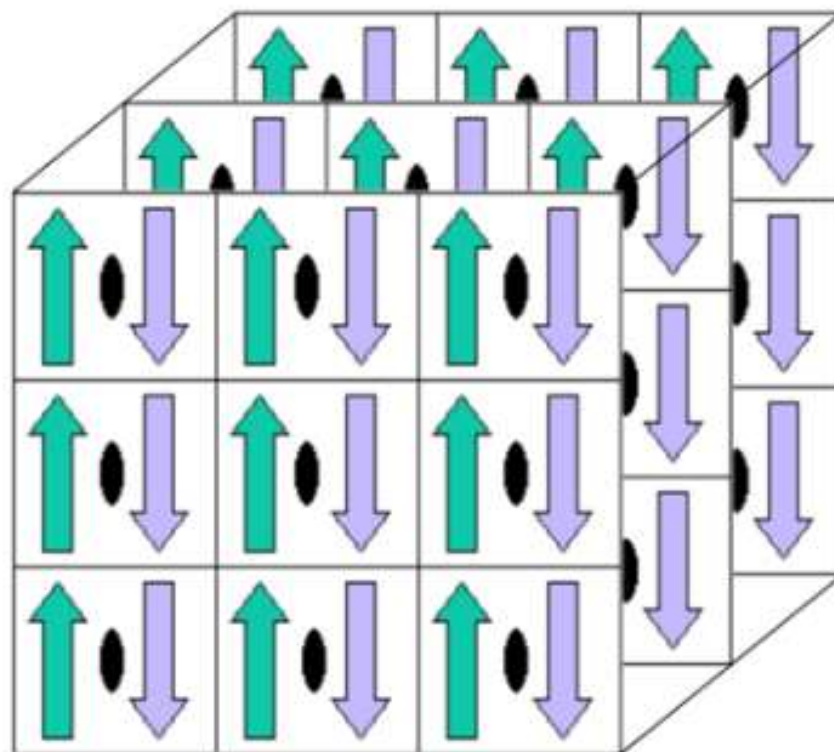




Asymmetric
Unit



Unit Cell



Entire Crystal

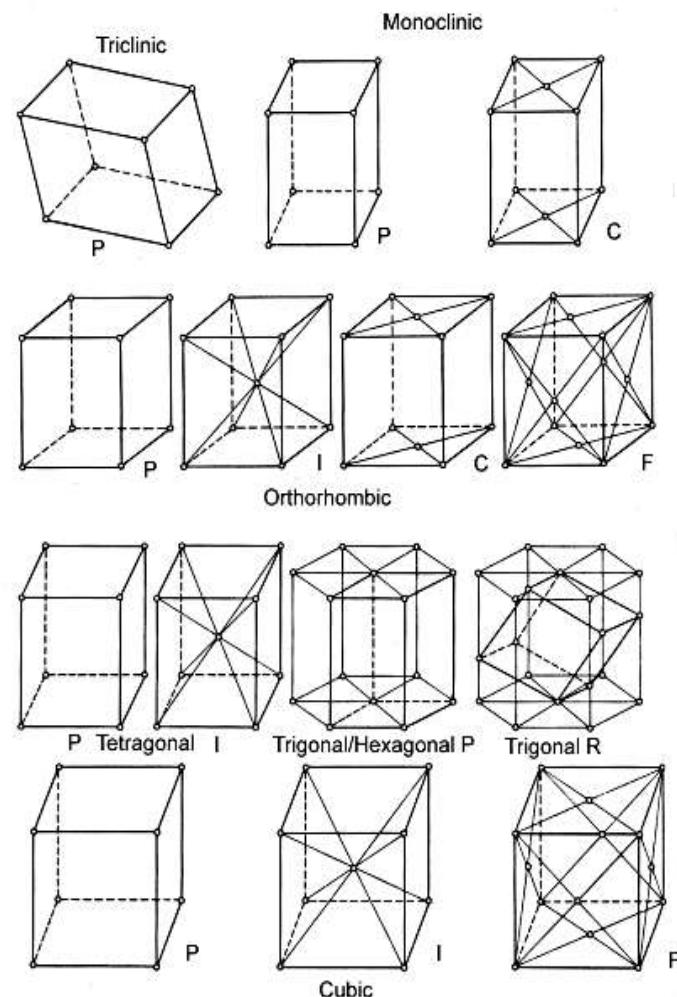
Crystal Systems

Crystal System	Bravais Type(s)	External Minimum Symmetry	Unit Cell Properties
Triclinic	P	None	a, b, c, α , β , γ
Monoclinic	P, C	One 2-fold axis, parallel b (b unique)	a, b, c, 90, β , 90
Orthorhombic	P, I, F	Three perpendicular 2-folds	a, b, c, 90, 90, 90
Tetragonal	P, I	One 4-fold axis, parallel c	a, a, c, 90, 90, 90
Trigonal	P, R	One 3-fold axis	a, a, c, 90, 90, 120
Hexagonal	P	One 6-fold axis	a, a, c, 90, 90, 120
Cubic	P, F, I	Four 3-folds along space diagonal	a, a, a, 90, 90, 90

Symmetry operations : 1, 2, 3, 4, 6, -1, -2, -3, -4, -6, m

Crystal System	Point groups	Laue Class	Patterson Symmetry
Triclinic	1, -1	-1	P-1
Monoclinic	2, m, 2/m	2/m	P2/m, C2/m
Orthorhombic	222, mm2, mmm	mmm	Pmmm, Cmmm, Fmmm, Immm
Tetragonal	4, -4, 4/m, 422, 4mm, -42m, 4/mmm	4/m, 4/mmm	P4/m, I4/m, P4/mmm, I4/mmm
Trigonal	3, -3, 32, 3m, -3m	-3, -3m	P-3, R-3, P-3m1, P-31m, R-3m
Hexagonal	6, -6, 6/m, 622, 6mm, -62m, 6/mmm	6/m, 6/mmm	P6/m, P6/mmm
Cubic	23, m-3, 432, -43m, m3m	m-3, m-3m	Pm-3, Im-3, F-3m, Pm-3m, Fm-3m, Im-3m

The 14 Bravais Lattices



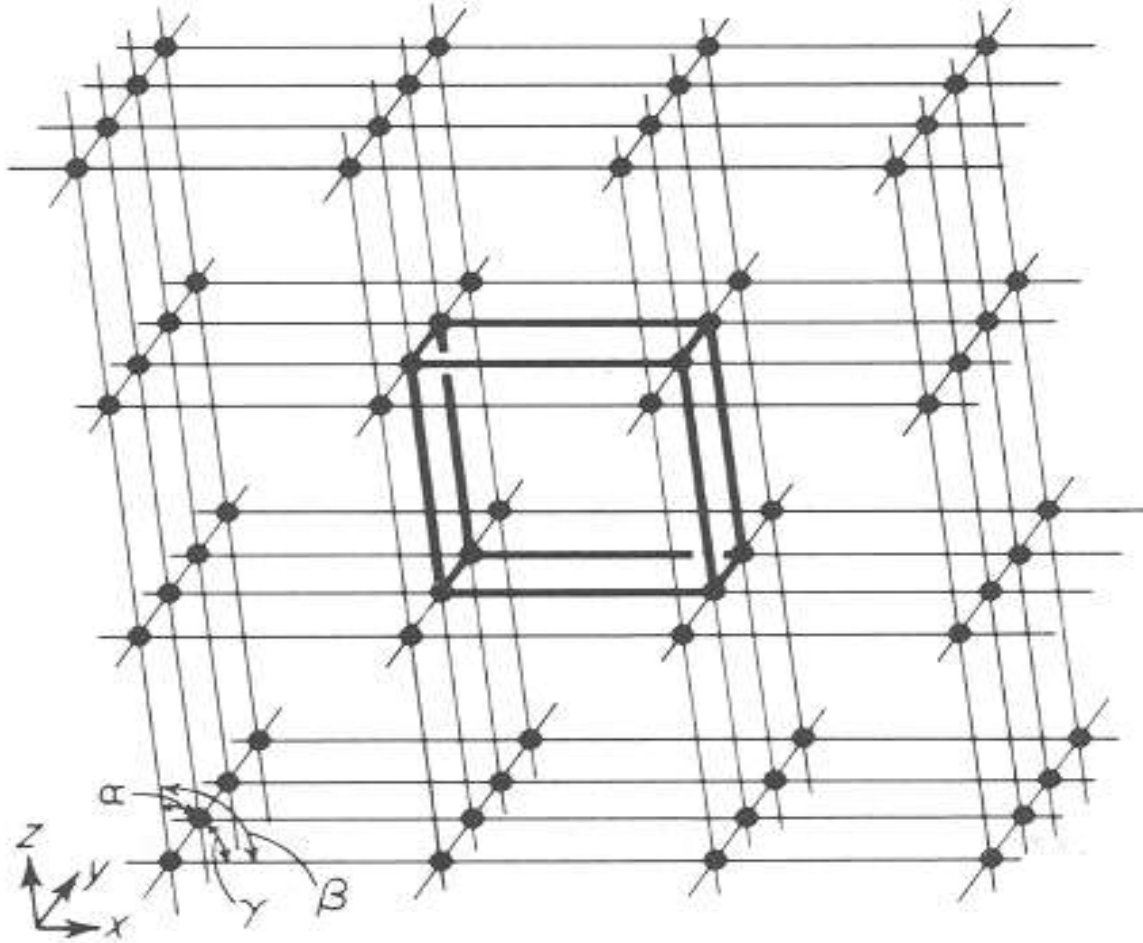
Notes

Note: Only 2-, 3-, 4-, and 6-fold rotations allowed

- Laue class corresponds to symmetry of reciprocal space (diffraction pattern)
- Patterson symmetry is Laue class plus allowed Bravais centering, i.e. centrosymmetric and symorphic

A **unit cell** is defined by its lattice constants:

a, b, c and **α, β, γ**

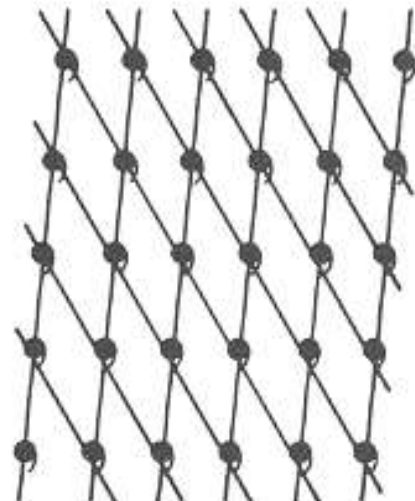
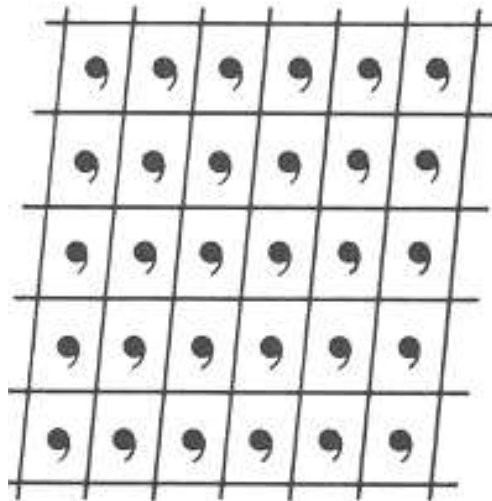
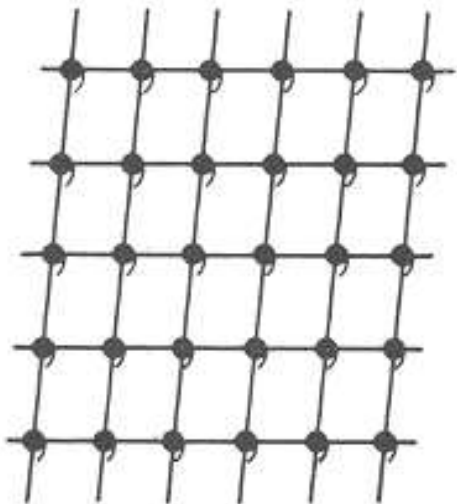


Three-dimensional lattice, showing unit cell (heavy lines).

How to identify “the” **unit cell** ?

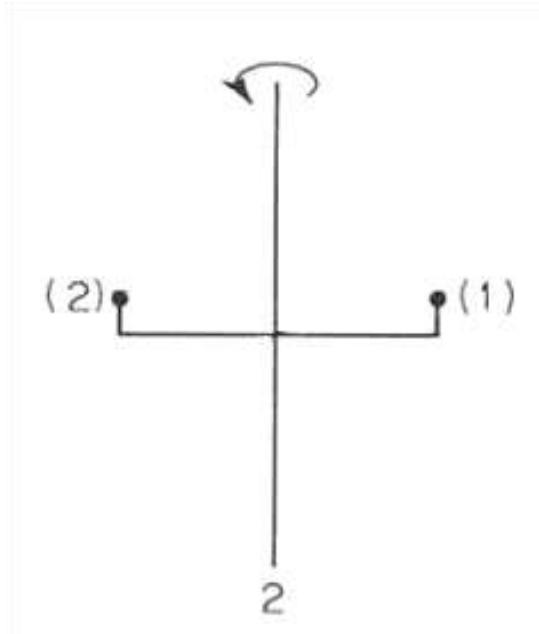


Regular two-dimensional array.

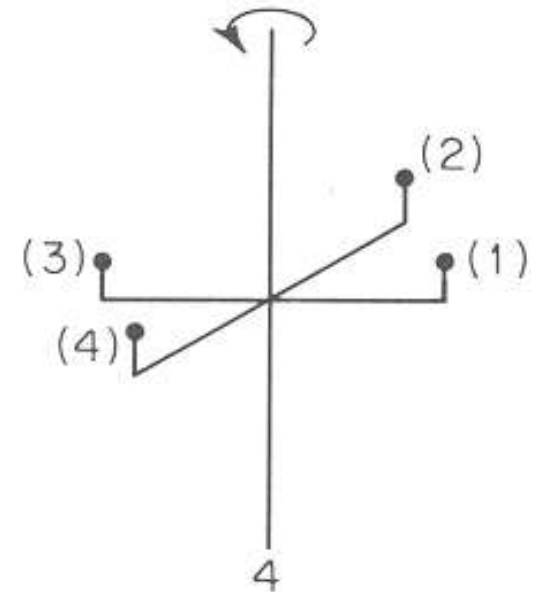


Three different grid systems referred to the array same array.

How to identify “the” **unit cell** ? **Role of Symmetry**



A 2-fold rotation axis.



A 4-fold rotation axis.

Rotation axes:

2-fold

3-fold

4-fold

6-fold

180°

120°

90°

60°

Translations:

$1/2$

$1/3$

$1/4$

$1/6$

Rot + Trans (Screw Axes) / Mirror / Inversion

How to identify “the” **unit cell** ? **Role of Symmetry**

Screw Axes

(rotation + translation)

2-fold screw

$180^\circ + 1/2$

3-fold screw

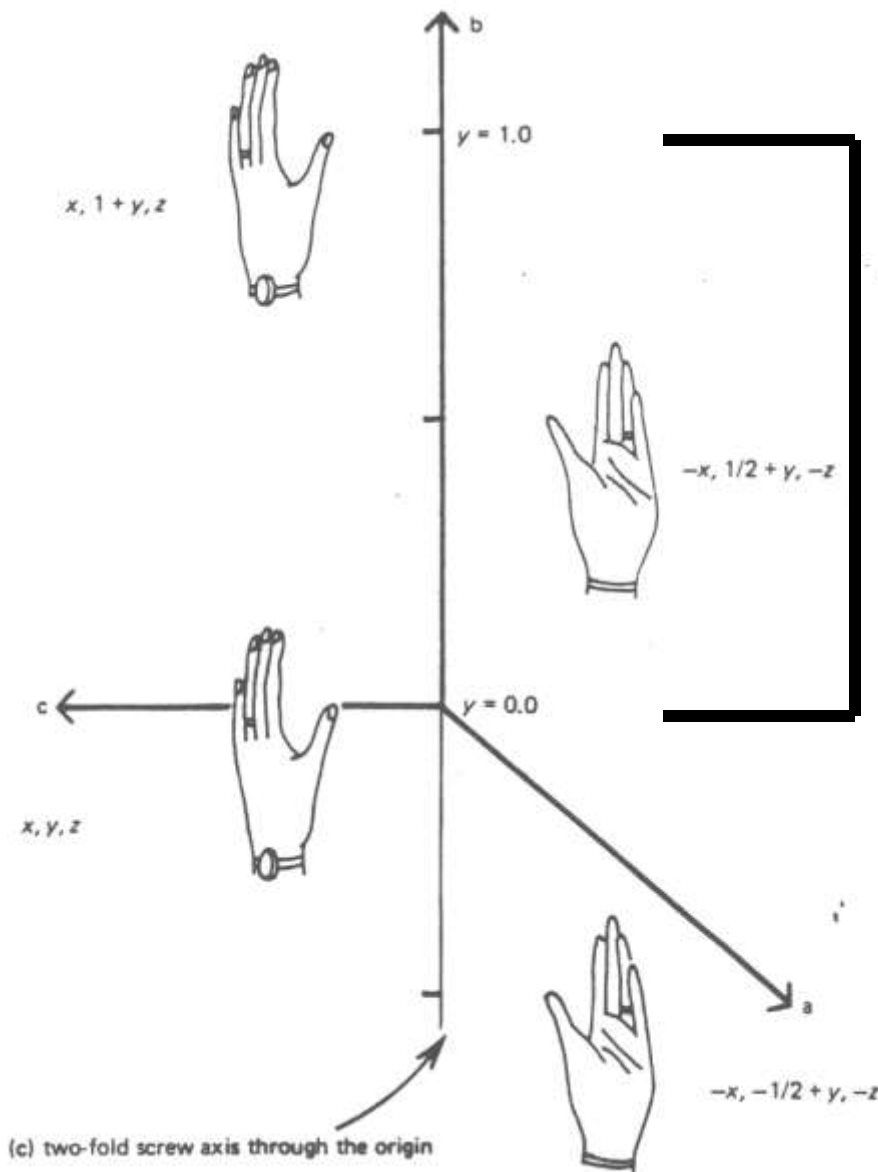
$120^\circ + 1/3$

4-fold screw

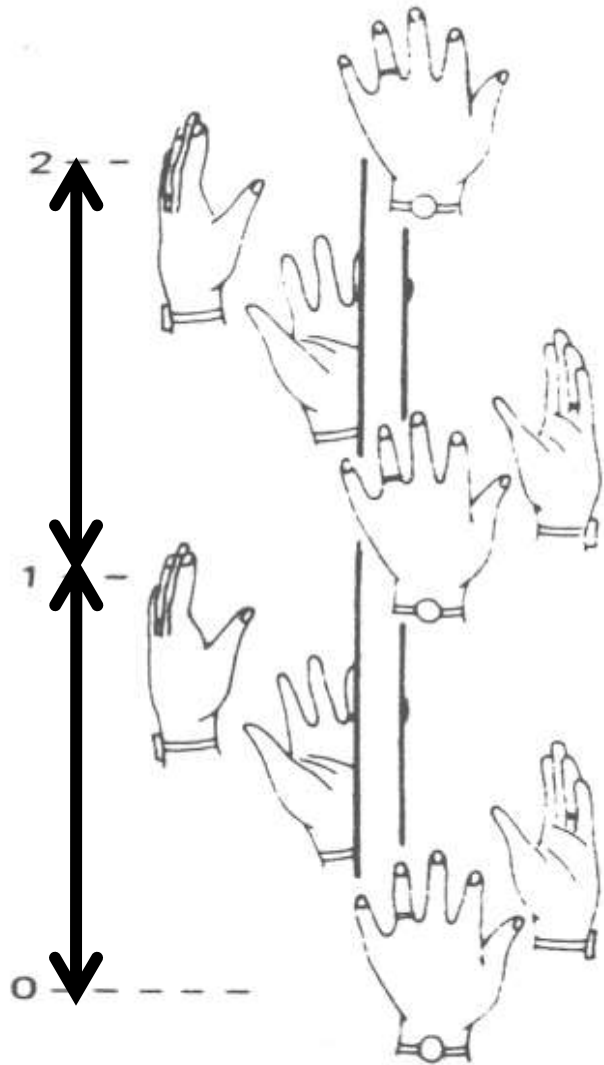
$90^\circ + 1/4$

6-fold screw

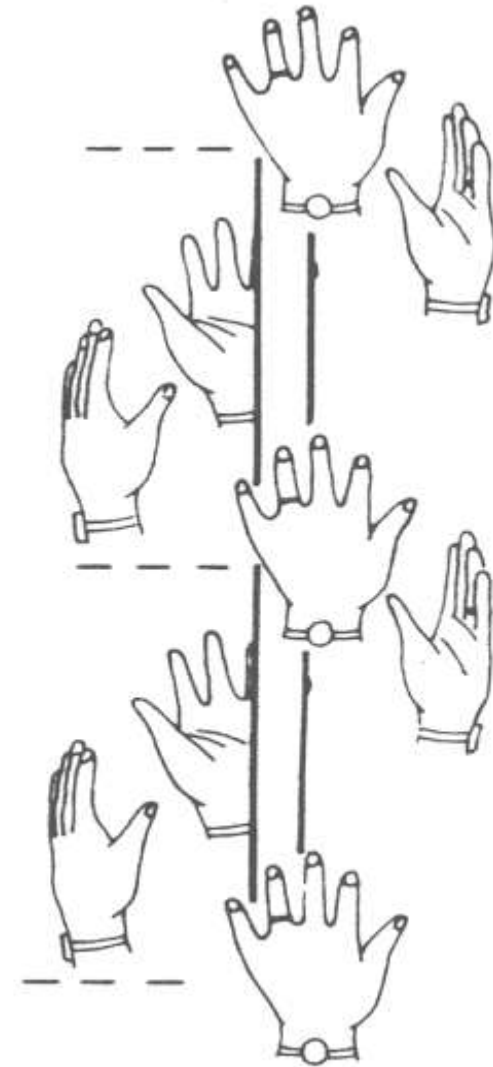
$60^\circ + 1/6$



4_1 and 4_3 Screw Axis



(d) 4_1 on left hand



4_3 on left hand

We will only look at examples having only 2-fold (2) or 2-fold screw (2_1) axes.

Unit Cell Selection is Based on Symmetry

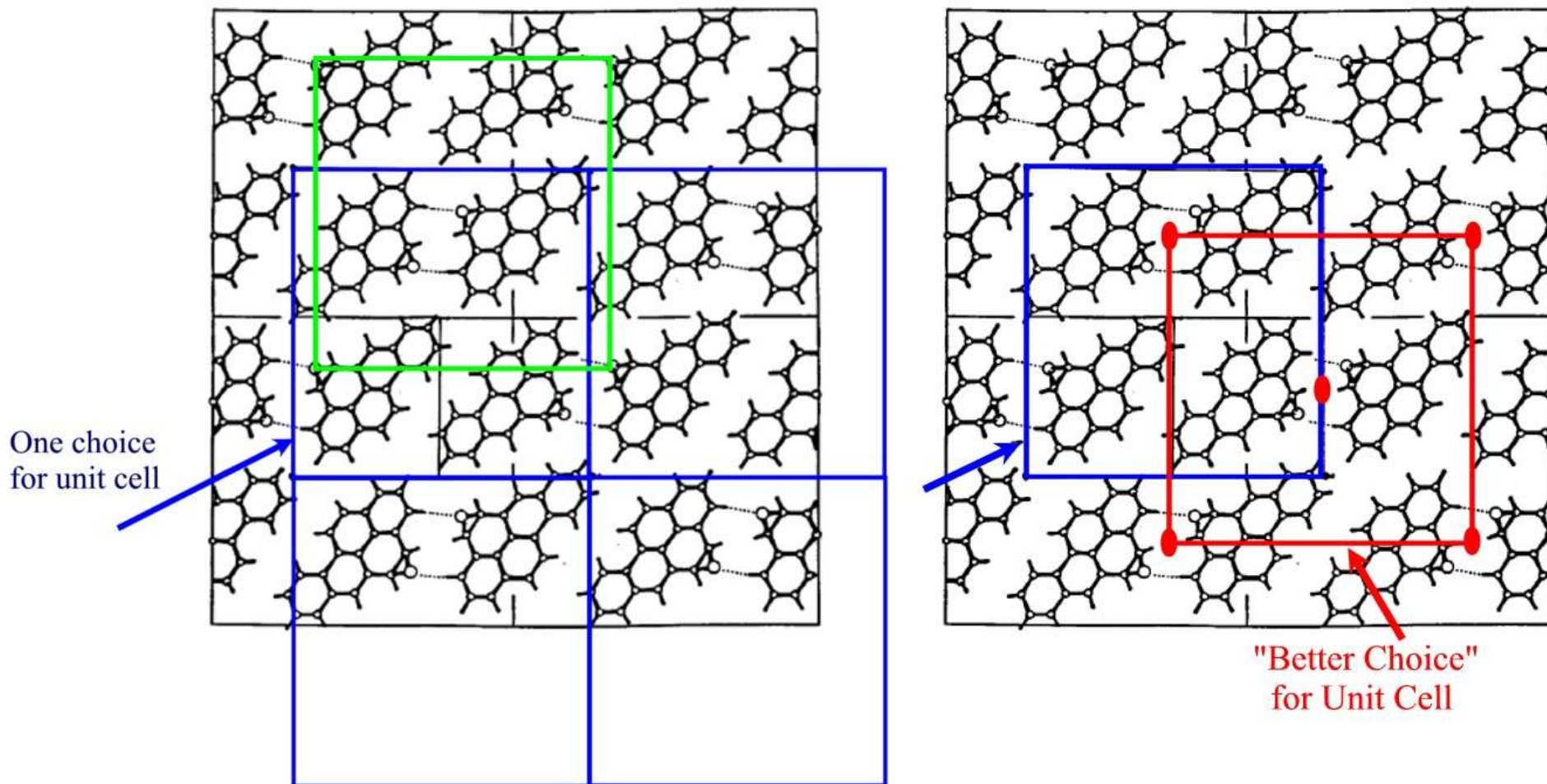
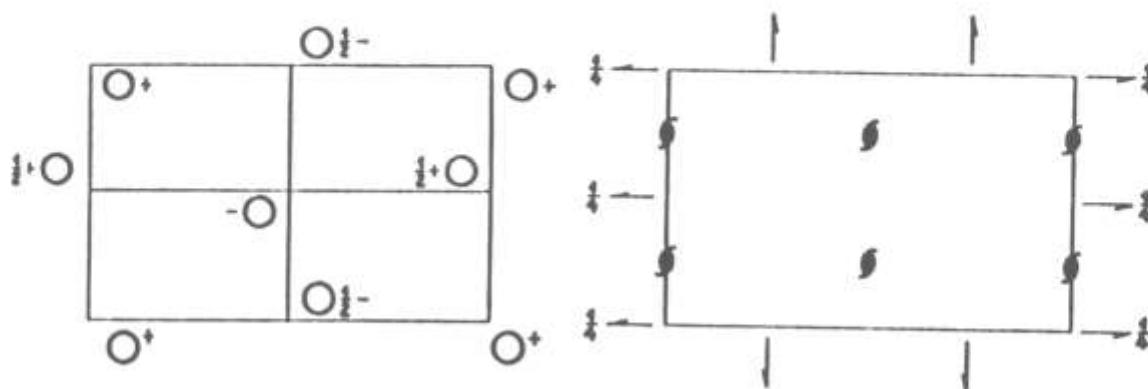


TABLE 16-5 The 65 "Biological" Space Groups

CRYSTAL SYSTEM	LAT-TICE	MINIMUM SYMMETRY OF UNIT CELL	UNIT CELL EDGES AND ANGLES ^a	DIFFRACTION PATTERN SYMMETRY ^b	SPACE GROUPS ^c
Triclinic	<i>P</i>	None	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$	$\bar{1}$	<i>P1</i>
Monoclinic	<i>P</i>	2-fold axis parallel to b	$a \neq b \neq c$	<i>2/m</i>	<i>P2</i> , <i>P2</i> ₁ <i>C2</i>
	<i>C</i>		$\alpha = \gamma = 90^\circ$ $\beta \neq 90^\circ$		
Orthorhombic	<i>P</i>	3 mutually perpendicular 2-fold axes	$a \neq b \neq c$	<i>mmm</i>	<i>P222</i> , <i>P2</i> ₁ <i>2</i> ₁ <i>2</i> ₁ , <i>P222</i> ₁ , <i>P2</i> ₁ <i>2</i> ₁ <i>2</i> ₁ <i>C222</i> , <i>C222</i> ₁ [<i>I222</i> , <i>I2</i> ₁ <i>2</i> ₁ <i>2</i> ₁] <i>F222</i>
	<i>C</i>		$\alpha = \beta = \gamma = 90^\circ$		
	<i>I</i>				
	<i>F</i>				
Tetragonal	<i>P</i>	4-fold axis parallel to c	$a = b \neq c$	<i>4/m</i>	<i>P4</i> , (<i>P4</i> ₁ , <i>P4</i> ₃), <i>P4</i> ₂
	<i>I</i>		$\alpha = \beta = \gamma = 90^\circ$	<i>4/mmm</i>	<i>I4</i> , <i>I4</i> ₁ <i>P422</i> , (<i>P4</i> ₁ <i>22</i> , <i>P4</i> ₃ <i>22</i>), <i>P4</i> ₂ <i>22</i> <i>P42</i> ₁ <i>2</i> , (<i>P4</i> ₁ <i>2</i> ₁ <i>2</i> , <i>P4</i> ₃ <i>2</i> ₁ <i>2</i>), <i>P4</i> ₂ <i>2</i> ₁ <i>2</i> <i>I422</i> , <i>I4</i> ₁ <i>22</i>
Trigonal/rhombohedral	<i>R</i> ^d	3-fold axis parallel to c	$a = b = c$	$\bar{3}$	<i>R3</i>
	<i>P</i> ^d		$\alpha = \beta = \gamma \neq 90^\circ$	$\bar{3}m$	<i>P3</i> , (<i>P3</i> ₁ , <i>P3</i> ₂) <i>R32</i> [<i>P321</i> , <i>P312</i>] [(<i>P3</i> ₁ <i>21</i> , <i>P3</i> ₂ <i>21</i>), (<i>P3</i> ₁ <i>12</i> , <i>P3</i> ₂ <i>12</i>)]
Hexagonal	<i>P</i>	6-fold axis parallel to c	$a = b \neq c$	<i>6/m</i>	<i>P6</i> , (<i>P6</i> ₁ , <i>P6</i> ₅)
			$\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$	<i>6/mmm</i>	<i>P6</i> ₃ , (<i>P6</i> ₂ , <i>P6</i> ₄) <i>P622</i> , (<i>P6</i> ₁ <i>22</i> , <i>P6</i> ₅ <i>22</i>) <i>P6</i> ₃ <i>22</i> , (<i>P6</i> ₂ <i>22</i> , <i>P6</i> ₄ <i>22</i>)
Cubic	<i>P</i>	3-fold axes along cube diagonals	$a = b = c$	<i>m3</i>	<i>P23</i>
	<i>I</i> <i>F</i>		$\alpha = \beta = \gamma = 90^\circ$		<i>P2</i> ₁ <i>3</i> [<i>I23</i> , <i>I2</i> ₁ <i>3</i>] <i>F23</i>
				<i>m3m</i>	<i>P432</i> , (<i>P4</i> ₁ <i>32</i> , <i>P4</i> ₃ <i>32</i>) <i>P4</i> ₂ <i>22</i> <i>I432</i> , <i>I4</i> ₁ <i>32</i> <i>F432</i> , <i>F4</i> ₁ <i>32</i>



Origin halfway between three pairs of non-intersecting screw axes

Number of positions,
Wyckoff notation,
and point symmetry

Co-ordinates of equivalent positions

Conditions limiting
possible reflections

4 a 1 $x, y, z; \frac{1}{2} - x, \bar{y}, \frac{1}{2} + z; \frac{1}{2} + x, \frac{1}{2} - y, \bar{z}; \bar{x}, \frac{1}{2} + y, \frac{1}{2} - z.$

$hkl:$
 $0kl:$
 $h0l:$
 $hk0:$

} No conditions

$h00: h=2n$
 $0k0: k=2n$
 $00l: l=2n$

Symmetry of special projections

(001) $pgg; a'=a, b'=b$

(100) $pgg; b'=b, c'=c$

(010) $pgg; c'=c, a'=a$

FIGURE 7.2 Part of a Page from "International Tables for X-Ray Crystallography," Volume I.

BPTI Space Group P212121



Asymmetric Unit

