

# X-Ray Crystallography

*“If a picture is worth a thousand words, then a macromolecular structure is priceless to a physical biochemist.” – van Holde*

Topics:

## 1. Protein Data Bank (PDB)

### 2. Image Formation

Resolution / Wavelength (Amplitude, Phase) / Light Microscopy / EM / X-ray / (NMR)

### 2. X-Ray Crystallography

- Crystal Growth – Materials / Methods
- Crystal Lattices - Lattice Constants / Space Groups / Asymmetric Unit
- X-ray Sources – Sealed Tube / Rotation Anode / Synchrotron
- Theory of Diffraction – Bragg’s Law / Reciprocal Space
- Data Collection – Methods / Detectors / Structure Factors
- Structure Solution – Phase Problem: MIR / MR / MAD
- Refinements and Models
- Analysis and presentation of results

## PDB Holdings List: 5-Apr-2005

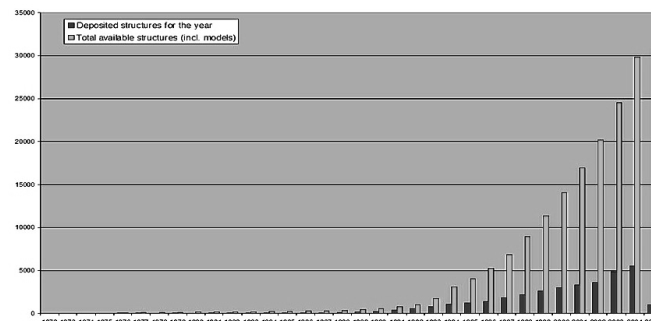
	X-ray Diffraction and other	Molecule Type				Total
		Proteins, Peptides, and Viruses	Protein/Nucleic Acid Complexes	Nucleic Acids	Carbohydrates	
Exp.		23942	1147	778	11	25978
Tech.	NMR	3721	111	648	2	4483
	Total	27663	1258	1427	13	30361

Please note that theoretical models have been removed, effective July 02, 2002, as per EDR policy.

16255 Structure Factor Files

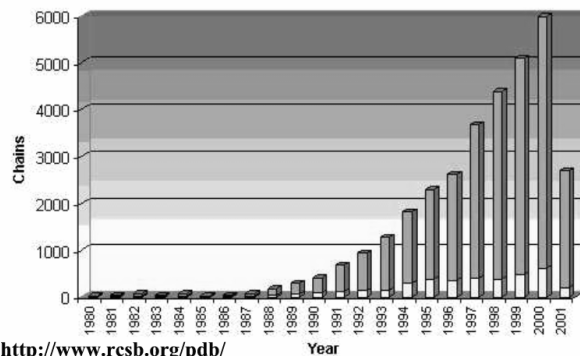
2464 NMR Restraint Files

### PDB Content Growth



## Protein Data Bank - 17,679 Structures - March 2002

X-ray - 14595  
NMR - 2706 "Old" Folds  
Theory - 378 "New" Folds



<http://www.rcsb.org/pdb/>

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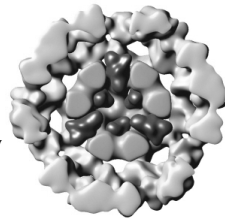
# Image Formation

Abbe (~1873): Limit Res.  $\sim \lambda/2$

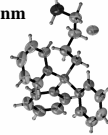


• Light Photography  
 $l \sim 400 - 700 \text{ nm}$

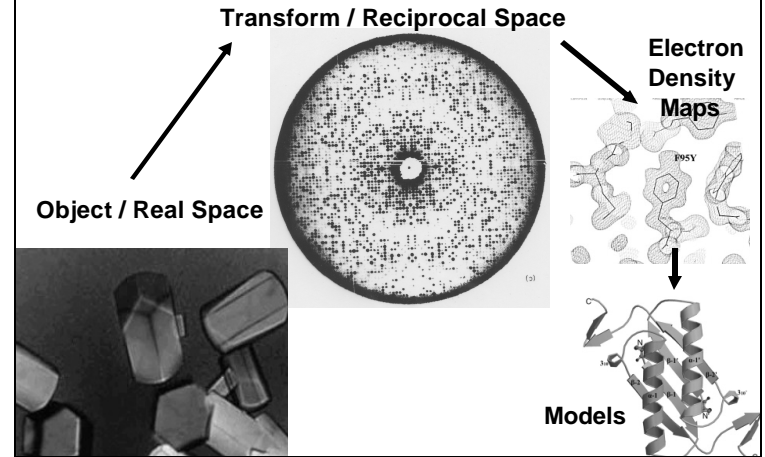
• Electron Microscopy  
 $l \sim 0.001 - 0.1 \text{ nm}$



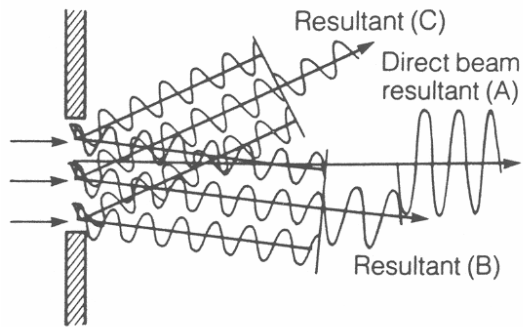
• X-Ray or NMR  
 $l \sim 0.1 \text{ nm}$



Object  $\longrightarrow$  Transform  $\longrightarrow$  Image



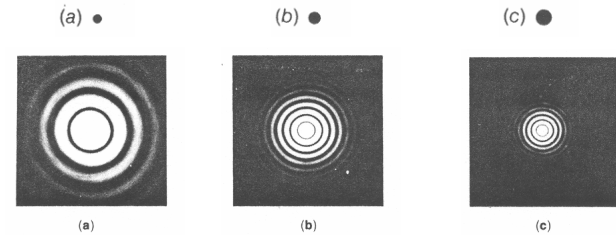
## Transforms / Reciprocal Space



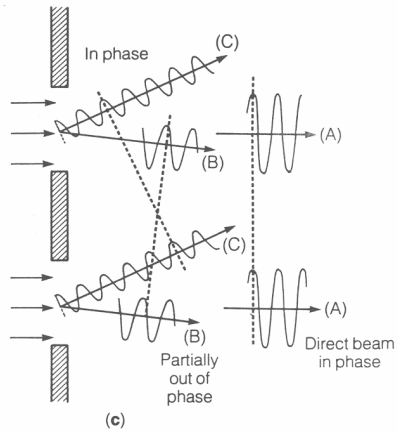
(b)

## Transforms / Reciprocal Space

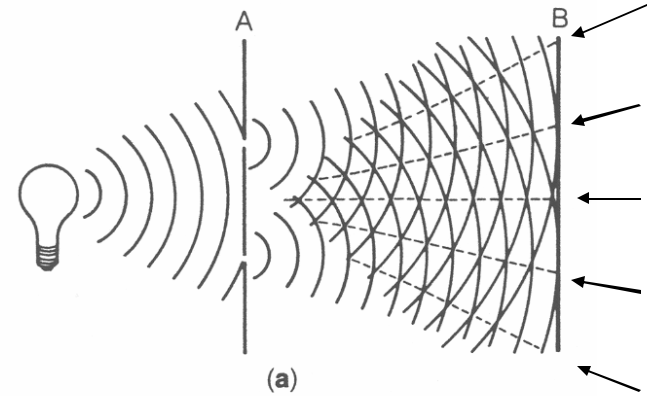
Different size holes



**Transforms / Reciprocal Space**

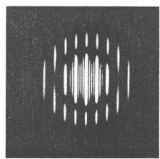


**Transforms / Reciprocal Space**

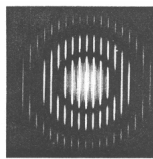


**Transforms / Reciprocal Space**

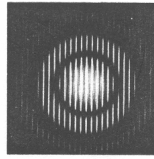
Five horizontal holes  
with various spacings



(j)



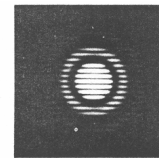
(k)



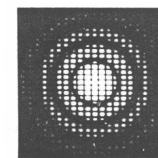
(l)

**Transforms / Reciprocal Space**

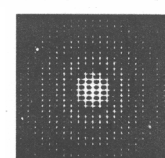
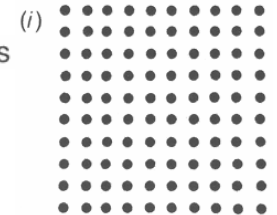
Vertical holes and nets of holes



(g)



(h)



(i)


Kevin Cowtan's Picture Book of Fourier Transforms - Netscape

File Edit View Go Communicator Help

Back Forward Reload Home Search Netscape Print Security Shop Stop

Bookmarks Location: <http://www.ysbl.york.ac.uk/~cowtan/fourier/fourier.html> What's Related

## Kevin Cowtan's Book of Fourier



This is a book of pictorial 2-d Fourier Transforms. These are particularly relevant to my own field of *X-ray crystallography*, but should be of interest to anyone involved in signal processing or frequency domain calculations.

**Contents:** <http://www.ysbl.york.ac.uk/~cowtan/fourier/fourier.html>

- [Introduction](#)
- [Book of Crystallography](#)
- [Duck Tales](#) and missing data
- [A Little Animal Magic](#) and cross phasing
- [A Tail of Two Cats](#) and image restoration
- [Animal Liberation](#) and free-sets.

• [The Gallery](#). Other interesting pictures.

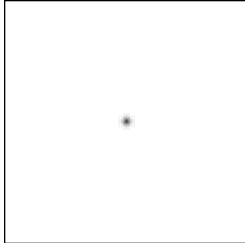
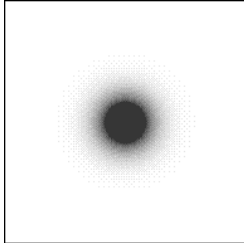
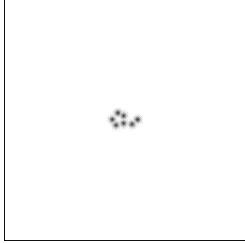
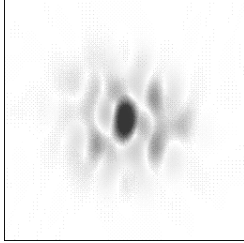
**Other topics:**

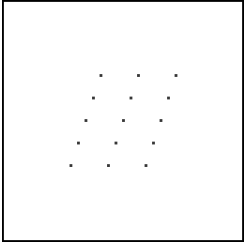
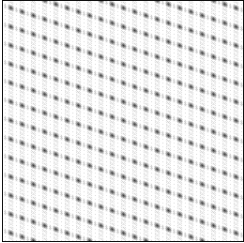
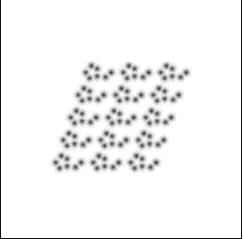
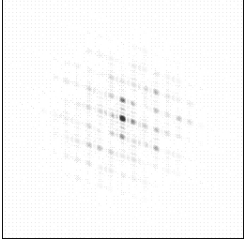
[The Interactive Structure Factor Tutorial](#): Learn about structure factors and maps.

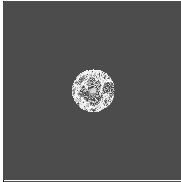
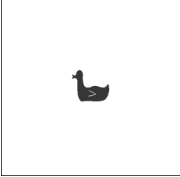
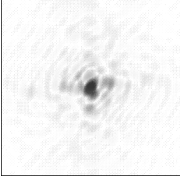
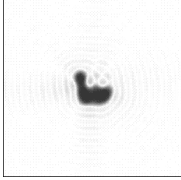
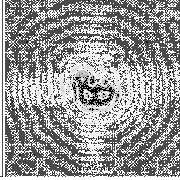
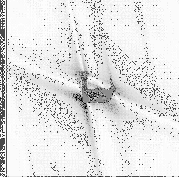
An introduction to crystallographic [Fourier transforms](#). The mathematical link between [Scattering theory](#) and Fourier theory. An explanation of the [convolution theorem](#).

**Teaching materials elsewhere**

Document: Done

Object / Real Space	Transform / Reciprocal Space
	
	

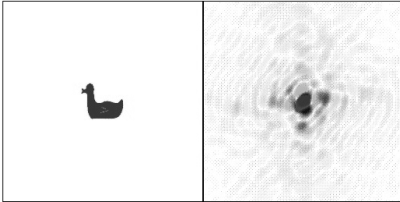
Object / Real Space	Transform / Reciprocal Space
	
	

Objects – Transforms and Image Formation	A Duck	Transform of a Duck
		
		

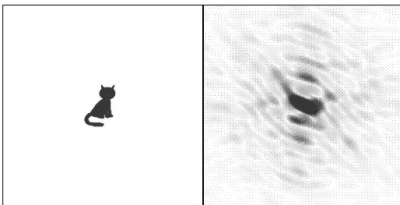
## Kevin Cowtan's Book of Fourier

<http://www.ytbl.york.ac.uk/~cowtan/fourier/fourier.html>

Here is our old friend, the Fourier Duck, and his Fourier transform:



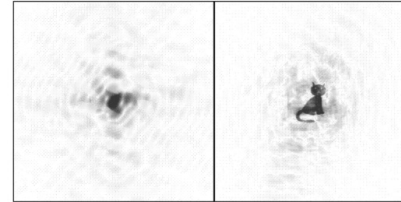
And here is a new friend, the Fourier Cat and his Fourier transform:



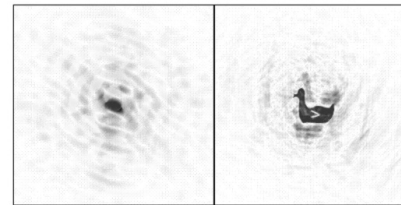
## Kevin Cowtan's Book of Fourier

<http://www.ytbl.york.ac.uk/~cowtan/fourier/fourier.html>

Duck Transform Amplitudes + Cat Phases

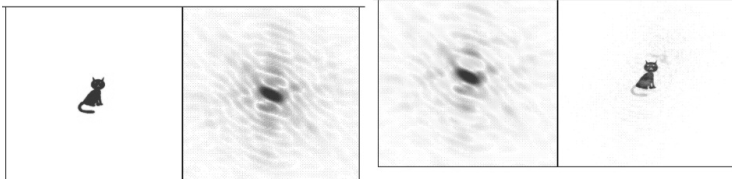


Cat Transform Amplitudes + Duck Phases



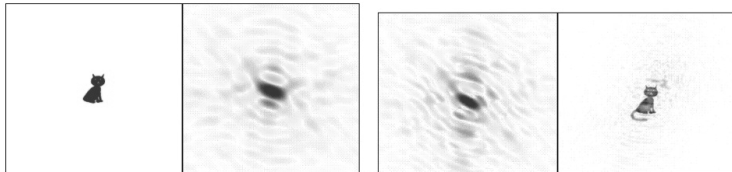
## Kevin Cowtan's Book of Fourier

<http://www.ytbl.york.ac.uk/~cowtan/fourier/fourier.html>



a) Cat - Cat Transform (Amplitudes only)  
b) Manx (tailless) Cat - Manx Transform

c) Cat Amplitudes + Manx Phases  
d) [ 2x(Cat Amplitudes) - Manx Amplitudes]  
+ Manx Phases



## X-Ray Crystallography

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Topics:

### 1. Image Formation

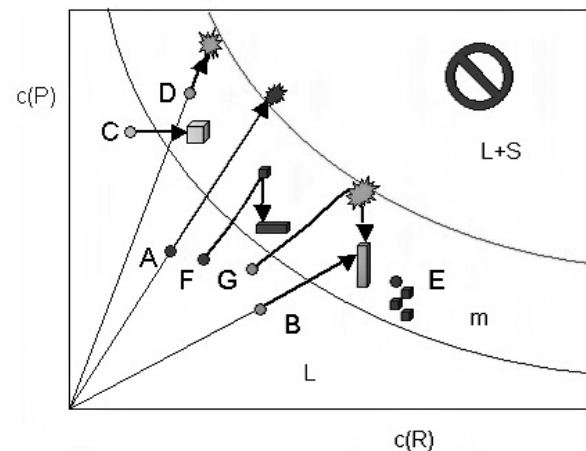
Resolution / Wavelength (Amplitude, Phase) / Light Microscopy / EM / Xray / NMR

### 2. X-Ray Crystallography

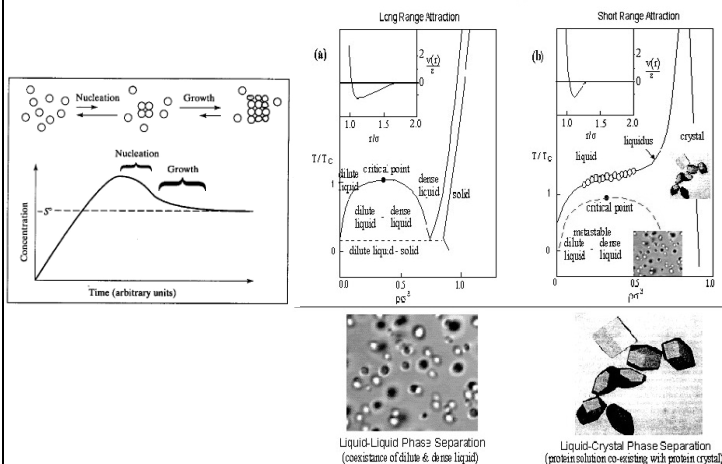
- a) Crystal Growth – Materials / Methods
- b) Crystal Lattices - Lattice Constants / Space Groups
- c) X-ray Sources – Sealed Tube / Rotation Anode / Synchrotron
- d) Theory of Diffraction – Bragg's Law / Reciprocal Space
- e) Data Collection – Methods / Detectors / Structure Factors
- f) Structure Solution – Phase Problem: MIR / MR / MAD
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## Variables that influence crystal growth

1. Nature of macromolecule – Purity and concentration of macromolecule
2. Nature and concentration of precipitant
3. pH / Temperature / Pressure
4. Level of reducing agent or oxidant
5. Substrates, coenzymes, and ligands / Metal ions
6. Preparation and storage of macromolecule / Proteolysis and fragmentation
7. Age of macromolecule / Degree of denaturation
8. Vibration and sound
9. Volume of crystallization sample
10. Seeding
11. Amorphous precipitate
12. Buffers
13. Cleanliness
14. Organism or species from which the macromolecule was isolated
15. Gravity, gradients and convection



## Protein Crystallization



## Types of Crystals

Welcome to the Home Page of Terese Bergfors  
"The Protein Crystallization Page" <http://xray.bmc.uu.se/~terese/>

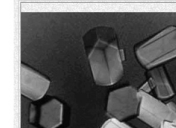
	<b>Sea urchins.</b> These often begin as spherulites (see previous tutorial) or grow from a spherulite. They are extremely thin needles clustered around a single nucleation site. Often they appear "fuzzy". The white arrows in this picture show three-dimensional crystals growing from independent nucleation sites in the same drop.
	<b>Needles.</b> Extremely thin needles growing from a single nucleation center. Since these needles are much longer, I would not call this a "sea urchin" any longer. Notice the huge 3-D crystal growing in the same drop.
	<b>Needles.</b> Still too many but at least they are single needles. The nucleation rate is too high which is why you have too many, and too thin. Try reducing the protein or precipitant concentration or both. Another method is to put a layer of oil over the reservoir in the vapor drop setup. (See Chayen, put reference here). See also the Tutorial 4 on seeding.
	<b>Plates.</b> Two-dimensional. Plates are usually considered an improvement over needles. These are growing from a single nucleation site and overlapping each other, which is far from optimal. Optimize to grow them as separately and thicker.
	<b>A three dimensional crystal.</b> But check the diffraction before you get out the champagne.

Welcome to the Home Page of Terese Bergfors  
"The Protein Crystallization Page"

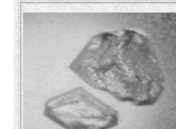
<http://xray.bmc.uu.se/~terese/>

## Tutorial 1. Appearances can be deceiving!

The first thing you should learn is that the appearance (habit, morphology, etc.) of your crystal is NOT what is important. Beautiful looking crystals may not diffract and really ugly crystals might diffract beautifully. The only definitive proof of a "GOOD" crystal is its diffraction pattern in the X-ray beam. Therefore, do not be misled by the appearance of your crystal—mount it and check it in the beam.



**Beauty is only skin deep.**  
Don't be fooled by appearances. These crystals may look nice, but they don't diffract.



These are ugly. In fact they are so ugly you probably wouldn't even bother to mount them but they diffract to 1.6Å.  
Most of the story don't always go by the appearances of your crystals. It is the X-ray diffraction pattern that counts.

With that said, you can now continue the tutorial on interpreting your crystallization drop.

## Common Compounds used in Crystallization

Ammonium or sodium sulfate  
 Sodium or ammonium citrate  
 Sodium or ammonium acetate  
 Magnesium sulfate  
 Cetyltrimethyl ammonium salts  
 Polyethylene glycol 400, 1000, 4000, 6000, 15,000 (now also 2,000, 8,000, etc.)

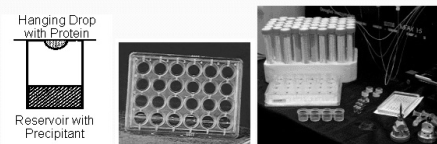
## Methods for protein crystallization

Batch crystallization (simply dump reagents together)  
 Liquid-liquid diffusion in a capillary tube  
 Vapor diffusion-the most successful method (hanging drop, sitting drop), typically using a Linbro plate. Equilibration occurs between the liquid and vapor phase.  
 Dialysis

## Hanging Drop Method - Crystal Screening

### The Experimental Setup

In order to obtain a crystal, the protein molecules must assemble into a periodic lattice. One starts with a solution of the protein with a fairly high concentration (2-50 mg/ml) and adds reagents that reduce the solubility close to spontaneous precipitation. By slow further concentration, and under conditions suitable for the formation of a few nucleation sites, small crystals **may** start to grow. Often very many conditions have to be tried to succeed. This is usually done by **initial screening**, followed by a systematic optimization of conditions. Crystals should be a few tenths of a mm in each direction to be useful for diffraction experiments.



Right: The hanging drop technique. Center: 24 such hanging drop experiments are set up in a Linbro plate. Right: A kit of different screening solutions, a set-up Linbro plate, dialysis buttons and a micro batch plate behind a goniometer head.

The most common setup to grow protein crystals is by the **hanging drop** technique. A few microliters of protein solution are mixed with an about equal amount of reservoir solution containing the precipitant. A drop of this mixture is put on a glass slide which covers the reservoir. As the protein/precipitant mixture in the drop is less concentrated than the reservoir solution (remember: we mixed the protein solution with the reservoir solution about 1:1), water evaporates from the drop into the reservoir. As a result the concentration of both protein and precipitant in the drop slowly increases, and crystals may form. There is a variety of other techniques available such as sitting drops, dialysis buttons, and gel and microbatch techniques. Robots are useful for automatic screening and optimization of crystallization conditions. We have implemented a web computing service of **Ernst Siggel's CRYSTOOL**, an inherently efficient random screen for crystallization conditions that you can customize. The main advantage is the small sample size a **crystallization robot** can handle reproducibly, but it needs some effort to set it up. Click here to learn more about the **IMPAX-II** robot.

## Hampton Crystal Screen Solutions

...

### Note:

A mini-screen can be set up from the most successful conditions. Those are indicated in the column labeled Miniscreen.

- [a nice list of detergents](#) commonly used in crystallization experiments.
- [pretty pictures](#)
- **CRYSTOOL**, efficient random screen made for you here on the WEB

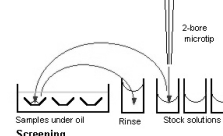


Tube #	SALT	BUFFER	Precipitant	Miniscreen	Tube #
1	0.02M Calcium Chloride	0.1M Na Acetate pH 4.6	30% w/v 2-methyl-2,4-pentanediol	Y	1
2	None	None	0.4M K <sub>2</sub> Na Tartrate tetrahydrate		2
3	None	None	0.4M Ammonium dihydrogen phosphate		3
4	None	0.1M Tris-HCl pH 8.5	2.0M Ammonium Sulfate	Y	4
5	0.2M Trisodium citrate	0.1M Na HEPES pH 7.5	30% w/v 2-methyl-2,4-pentanediol		5
6	0.2M Magnesium chloride	0.1M Tris-HCl pH 8.5	30% w/v PEG 4000		6
7	None	0.1M Na Cacodylate pH 6.5	1.4M Sodium acetate trihydrate		7
8	0.2M Trisodium citrate	0.1M Na Cacodylate pH 6.5	30% v/v 2-propanol		8
9	0.2M Ammonium acetate	0.1M Na Citrate pH 5.6	30% w/v PEG 4000	Y	9
10	0.2M Ammonium acetate	0.1M Na Acetate pH 4.6	30% w/v PEG 4000	Y	10
11	None	0.1M Na Citrate pH 5.6	1.0M Ammonium dihydrogen phosphate		11
12	0.2M Magnesium chloride	0.1M Na HEPES pH 7.5	30% v/v 2-propanol		12
13	0.2M Trisodium citrate	0.1M Tris-HCl pH 8.5	30% w/v PEG 400		13
14	0.2M Calcium Chloride	0.1M Na HEPES pH 7.5	28% v/v PEG 400	Y (best)	14
15	0.2M Ammonium acetate	0.1M Na Cacodylate pH 6.5	30% w/v PEG 8000		15
16	None	0.1M Na HEPES pH 7.5	1.5M Lithium sulfate monohydrate	Y	16
17	0.2M Lithium sulfate	0.1M Tris-HCl pH 8.5	30% w/v PEG 4000	Y (2nd best)	17

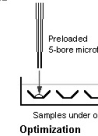
## Using Oryx 6 for Crystallization with Microbatch

Microbatch operation is identical to **IMPAX I-S**

**Very Fast - 240 wells/hour**  
 more than 50 wells/hour for screening



**Uses little protein**  
 0.1 - 1 µl per well



**Microbatch screening finds more leads**  
 than VD in a given time

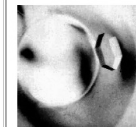
	Automated MB	Manual VD
Proteins screened	6	6
Screening solutions	48	48
Runs	3	1
Volume per well	1 + 1 µl	4 + 4 µl
Total protein used	864 µl	1152 µl
Operator time	3 hr	24 hr
Crystallization conditions found	43	41
Unique conditions	17	15

**Large diffracting**

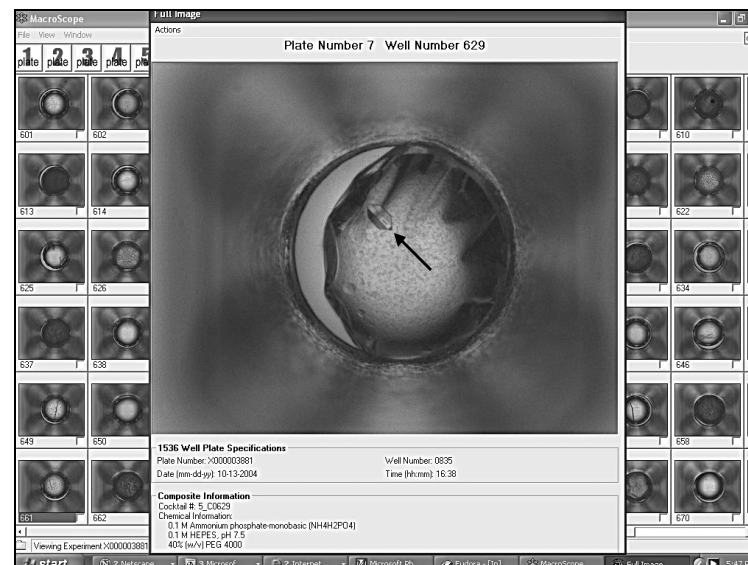
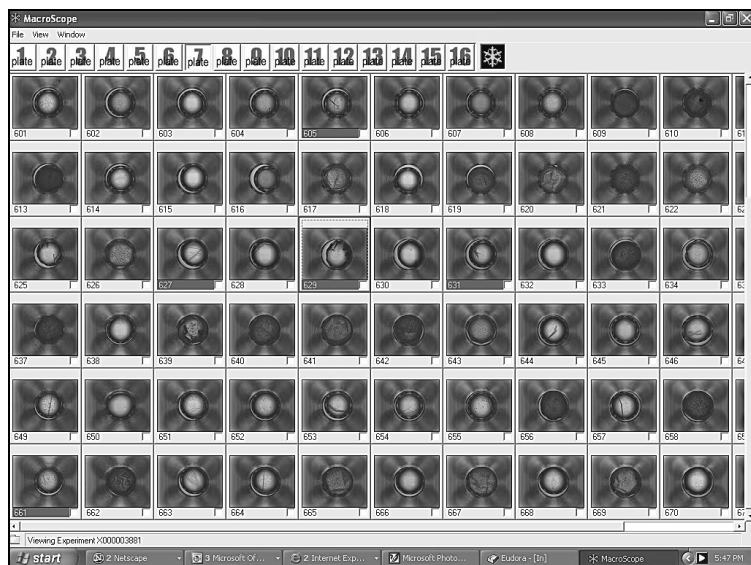


10 crystals of ribitol dehydrogenase from *Escherichia coli* collected from 10 crystals which were unstable. Courtesy of R. Crowl. Acta Crystallographica B 52 (1996) 4 pp 176-178. <http://www.douglas.co.uk/bio3.htm>

**crystals**



10 crystals of alcohol dehydrogenase from *Escherichia coli* bacteria. Courtesy of Y. Rothbar. Acta Crystallographica B 52 (1996) 1 pp 882-886. <http://www.douglas.co.uk/bio3.htm>



**HisJ - His complex:**

**Tetanus Toxin C Fragment:**

[http://www.ccp14.ac.uk/ccp/web-mirrors/llnlrupp/crystal\\_lab/Crys\\_lab.html](http://www.ccp14.ac.uk/ccp/web-mirrors/llnlrupp/crystal_lab/Crys_lab.html)

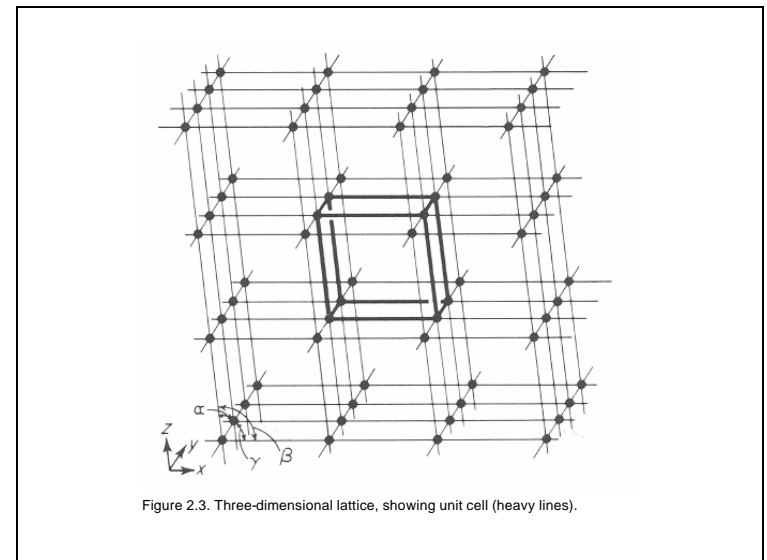
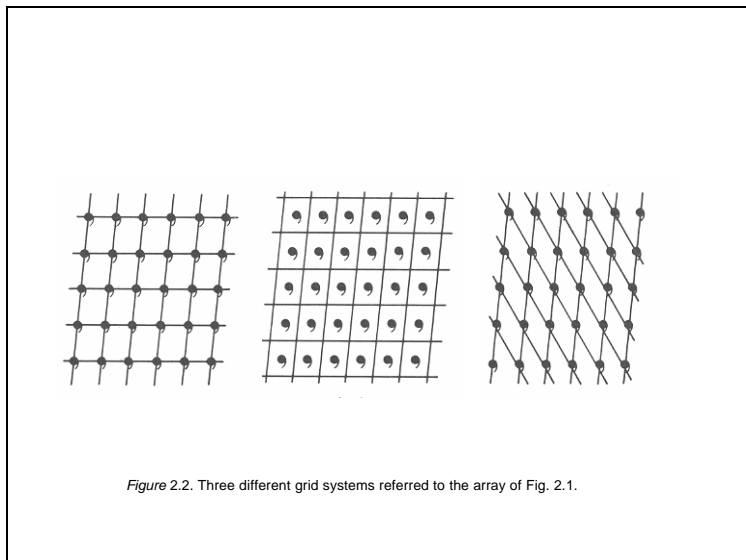
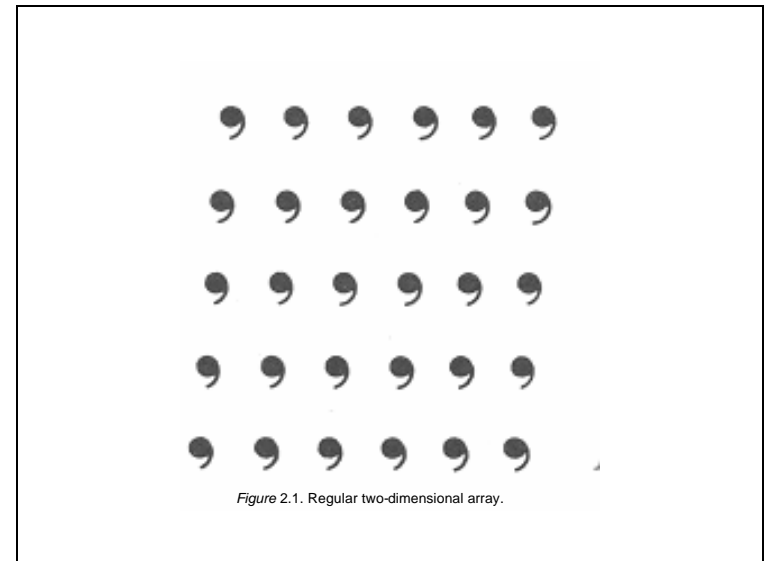
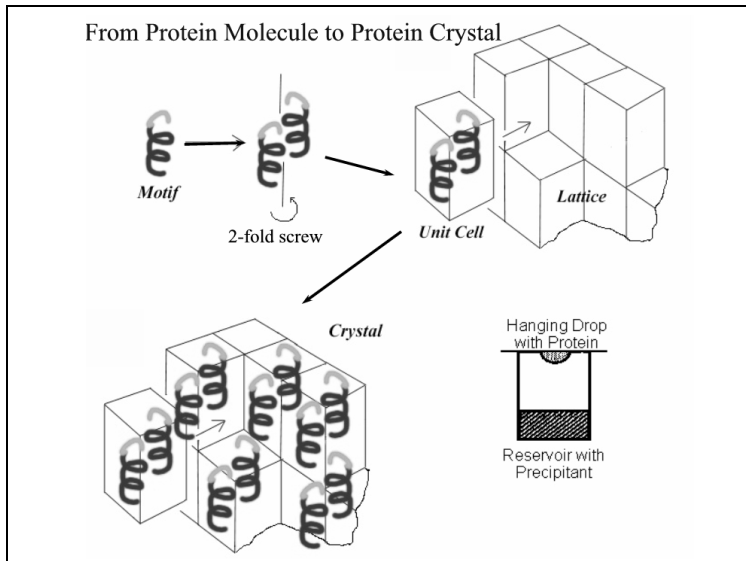
## X-Ray Crystallography

*“If a picture is worth a thousand words, then a macromolecular structure is priceless to a physical biochemist.” – van Holde text*

**Topics:**

1. Image Formation
  - Resolution / Wavelength (Amplitude, Phase) / Light Microscopy / EM / X-ray / NMR
2. X-Ray Crystallography
  - a) Crystal Growth – Materials / Methods
  - b) What is a Crystal: Lattices - Lattice Constants / Space Groups / Asymmetric Unit
  - c) X-ray Sources – Sealed Tube / Rotation Anode / Synchrotron
  - d) Theory of Diffraction – Bragg’s Law / Reciprocal Space
  - e) Data Collection – Methods / Detectors / Structure Factors
  - f) Structure Solution – Phase Problem: MIR / MR / MAD
  - h) Refinements and Models
  - i) Analysis and presentation of results





### Unit Cell Selection is Based on Symmetry

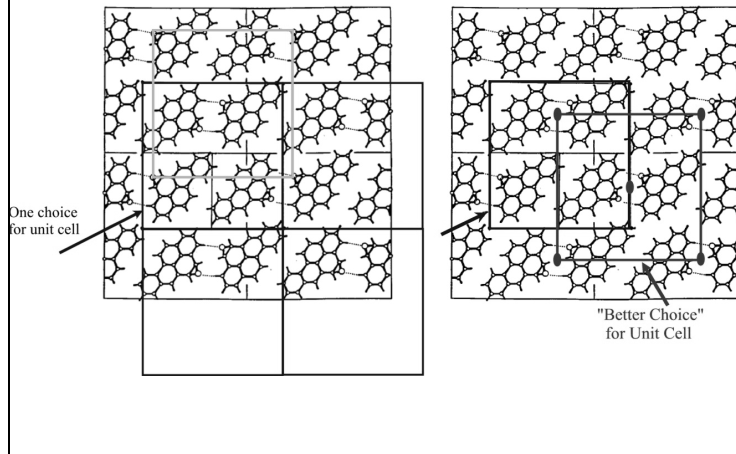


TABLE 16-1 Examples of Cyclic Point Groups

POINT GROUP <sup>a</sup>	H-M	NUM- BER OF ASYM- METRIC UNITS	CONTACT TYPE <sup>b</sup>	SYMMETRY ELEMENTS	EXAMPLES <sup>c</sup>
$C_1$	1	1	none		 Any polypeptide chain  Formic acid
$C_2$	2	2	$i$		 Hemoglobin  Hydrogen peroxide

### Examples of Cubic Point Groups

POINT GROUP <sup>a</sup>	H-M	NUM- BER OF ASYM- METRIC UNITS	CONTACTS <sup>b</sup> MAXI- MUM	CONTACTS <sup>b</sup> MINI- MUM	SYMMETRY ELEMENTS	EXAMPLES
T	23	12	hi	hi	 3 2-fold 4 3-fold	Aspartate- $\beta$ -decarboxylase <sup>d</sup> Neopentane <sup>d</sup> 
O	432	24	hii	hh or hi	 6 2-fold 4 3-fold 3 4-fold	Dihydropolyl transuccinylase <sup>d</sup>  Spherical subunit model  Electron density at low resolution

### Crystal Systems

Crystal System	Bravais Type(s)	External Minimum Symmetry	Unit Cell Properties
Triclinic	P	None	a, b, c, $\alpha$ , $\beta$ , $\gamma$
Monoclinic	P, C	One 2-fold axis, parallel b (b unique)	a, b, c, 90, $\beta$ , 90
Orthorhombic	P, I, F	Three perpendicular 2-folds	a, b, c, 90, 90, 90
Tetragonal	P, I	One 4-fold axis, parallel c	a, a, c, 90, 90, 90
Trigonal	P, R	One 3-fold axis	a, a, c, 90, 90, 120
Hexagonal	P	One 6-fold axis	a, a, c, 90, 90, 120
Cubic	P, F, I	Four 3-folds along space diagonal	a, a, a, 90, 90, 90

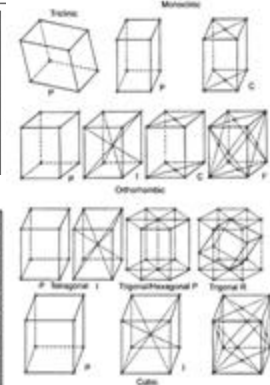
Symmetry operations: 1, 2, 3, 4, 6, m

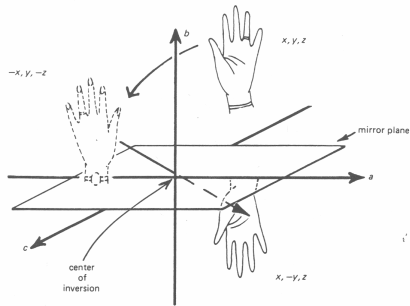
Crystal System	Point groups	Lattice Class	Pattern Symmetry
Triclinic	1, $\bar{1}$	$\bar{1}$	P-1
Monoclinic	2, m, 2/m	2/m	P2/m, C2/m
Orthorhombic	222, mm2, mmm	mmm	Fmmm, Cmmm, Fmmm, Immm
Tetragonal	4, $\bar{4}$ , 4/m, 422, 4mm, $\bar{4}2m$ , 4/mmm	4/m, 4/mmm	P4/m, I4/m, P4/mmm, I4/mmm
Trigonal	3, $\bar{3}$ , 32, 3m, $\bar{3}m$	$\bar{3}$ , $\bar{3}m$	P-3, R-3, P-3m1, P-31m, R-3m
Hexagonal	6, $\bar{6}$ , 6/m, 622, 6mm, $\bar{6}2m$ , 6/mmm	6/m, 6/mmm	P6/m, P6/mmm
Cubic	23, m-3, 432, $\bar{4}3m$ , m-3m	m-3, m-3m	Fm-3, Im-3, Fm-3m, Pm-3m, Ih-3m

#### Notes

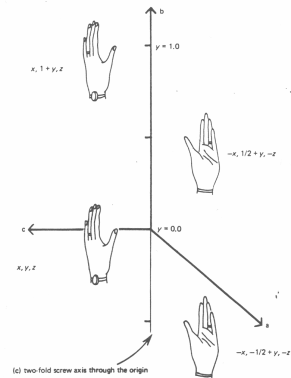
- Lattice class corresponds to symmetry of reciprocal space (diffraction pattern)
- Pattern symmetry is Lattice class plus allowed Bravais centering, i.e. centrosymmetric and acentric

### The 14 Bravais Lattices





(b) two-fold rotatory-inversion axis or mirror plane



(c) two fold screw axis through the origin

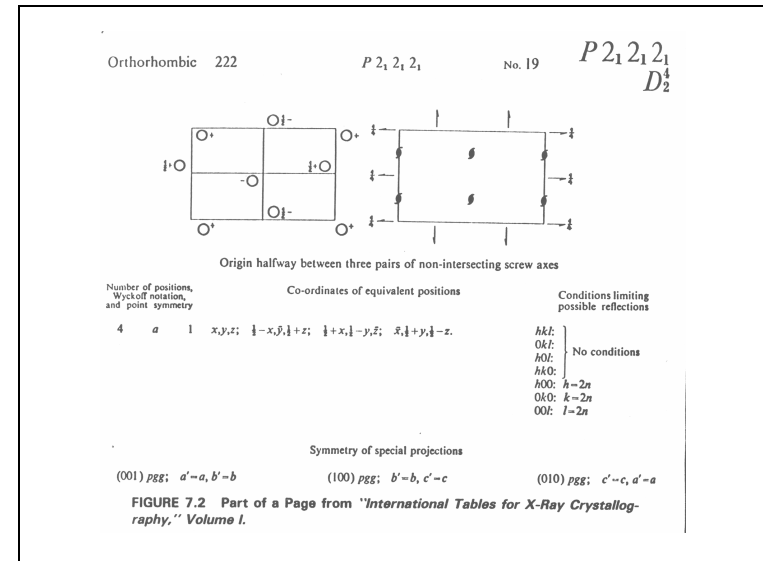
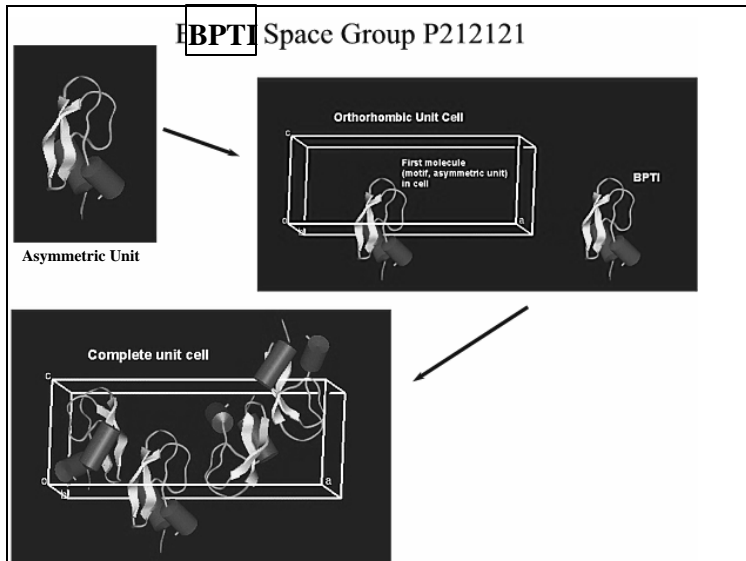
TABLE 3.4 Space Groups in Standard Orientations\*

System	Point Group	Space Group						Fraction	
Triclinic	1	P1						1/2	
Monoclinic	2	P2	P2 <sub>1</sub>	C2				1/4	
	m	Pm	Pc	Cm	Cc	P2 <sub>1</sub> /c	C2/c		
	2/m	P2/m	P2 <sub>1</sub> /m	C2/m	P2 <sub>1</sub> /c	C2/c			
Orthorhombic	222	P222	P2 <sub>1</sub> 2 <sub>1</sub> 2	P2 <sub>1</sub> 2 <sub>1</sub> 2	P2 <sub>1</sub> 2 <sub>1</sub> 2	C222 <sub>1</sub>	C222 <sub>2</sub>	1/8	
	mm2	F222	F222	F2 <sub>1</sub> 2 <sub>1</sub> 2	F2 <sub>1</sub> 2 <sub>1</sub> 2	F222	F222		
		Pmm2	Pmc2 <sub>1</sub>	Pnc2	Pma2	Pca2 <sub>1</sub>	Pmc2	Pnc2	
	mmm	Pmn2 <sub>1</sub>	Pma2	Pna2	Cmm2	Cma2	Cmc2	Cnc2	
		Cmc2	Amn2	Abn2	Amn2	Abn2	Fmm2		
		Fdd2	Fmm2	Fdn2	Fmm2	Fdn2			
		Pmmn	Pmna	Pbcm	Pban	Pbna	Pbcm	Pban	
		Pmna	Pca	Pbam	Pca	Pbcm	Pban	Pmna	
		Pmna	Pbcn	Pbca	Pbna	Cmcm	Cmca	Fddd	
	Cmmm	Cccm	Cmma	Ccca	Fmmm				
Innm	Ibam	Ibca	Ibca	Ibca					
Tetragonal	4	P4	P4 <sub>1</sub>	P4 <sub>2</sub>	P4 <sub>3</sub>	I4	I4 <sub>1</sub>	1/8	
	4	P4	P4 <sub>1</sub>	P4 <sub>2</sub>	P4 <sub>3</sub>	I4	I4 <sub>1</sub>		
	4/m	P4/m	P4 <sub>1</sub> /m	P4 <sub>2</sub> /m	P4 <sub>3</sub> /m	I4/m	I4 <sub>1</sub> /a		
	422	P422	P4 <sub>1</sub> 2 <sub>1</sub> 2	P4 <sub>2</sub> 22	P4 <sub>3</sub> 22	P4 <sub>2</sub> 22	P4 <sub>1</sub> 2 <sub>1</sub> 2	1/16	
	4mm	P4 <sub>2</sub> 22	P4 <sub>1</sub> 2 <sub>1</sub> 2	I422	I422	P4 <sub>2</sub> 22	P4 <sub>1</sub> 2 <sub>1</sub> 2		
		P4 <sub>1</sub> mm	P4 <sub>2</sub> mm	P4 <sub>1</sub> cm	P4 <sub>2</sub> cm	P4 <sub>1</sub> cc	P4 <sub>2</sub> cc		
	42m	P4 <sub>1</sub> mc	P4 <sub>2</sub> mc	I4mm	I4cm	I4 <sub>1</sub> md	I4 <sub>2</sub> cd		
		P4 <sub>1</sub> m	P4 <sub>2</sub> m	P4 <sub>1</sub> m	P4 <sub>2</sub> m	P4 <sub>1</sub> m	P4 <sub>2</sub> m		
	4/mmm	P4 <sub>1</sub> mm	P4 <sub>2</sub> mm	P4 <sub>1</sub> mm	P4 <sub>2</sub> mm	P4 <sub>1</sub> mm	P4 <sub>2</sub> mm		
		P4 <sub>1</sub> mbc	P4 <sub>2</sub> mbc	P4 <sub>1</sub> mbc	P4 <sub>2</sub> mbc	P4 <sub>1</sub> mbc	P4 <sub>2</sub> mbc		
	P4 <sub>1</sub> amd	P4 <sub>2</sub> amd	P4 <sub>1</sub> amd	P4 <sub>2</sub> amd	P4 <sub>1</sub> amd	P4 <sub>2</sub> amd			
Trigonal/rhombohedral	3	P3	P3 <sub>1</sub>	P3 <sub>2</sub>	R3			1/6	
	3	P3	P3 <sub>1</sub>	P3 <sub>2</sub>	R3				
	32	R32	P321	P3 <sub>1</sub> 12	P3 <sub>2</sub> 12	P3 <sub>1</sub> 12	P3 <sub>2</sub> 12	1/12	
Hexagonal	3m	P3m1	P31m	P3c1	P31c	R3m	R3c		
	3m	P31m	P31c	P3c1	P31c	R3m	R3c		
	6	P6	P6 <sub>1</sub>	P6 <sub>2</sub>	P6 <sub>3</sub>	P6 <sub>4</sub>	P6 <sub>5</sub>	1/12	
Cubic	6/m	P6/m	P6 <sub>1</sub> /m						
	622	P622	P6 <sub>1</sub> 22	P6 <sub>2</sub> 22	P6 <sub>3</sub> 22	P6 <sub>4</sub> 22	P6 <sub>5</sub> 22	1/24	
	6mm	P6mm	P6 <sub>1</sub> mm	P6 <sub>2</sub> mm	P6 <sub>3</sub> mm	P6 <sub>4</sub> mm	P6 <sub>5</sub> mm		
	6m2	P6m2	P6 <sub>1</sub> m2	P6 <sub>2</sub> m2	P6 <sub>3</sub> m2	P6 <sub>4</sub> m2	P6 <sub>5</sub> m2		
	6/mmm	P6/mmm	P6 <sub>1</sub> /mmm	P6 <sub>2</sub> /mmm	P6 <sub>3</sub> /mmm	P6 <sub>4</sub> /mmm	P6 <sub>5</sub> /mmm		
	23	P23	P23	I23	P23	I23	P23	1/24	
Cubic	m3	Pm3	Pn3	Fm3	Fd3	Im3	Pa3		
	432	P432	P4 <sub>3</sub> 32	F432	F4 <sub>3</sub> 32	I432	P4 <sub>3</sub> 32	1/48	
	43m	P43m	P4 <sub>3</sub> m	F43m	F4 <sub>3</sub> m	I43m	P4 <sub>3</sub> m		
	m3m	Pm3m	Pn3m	Fm3m	Fd3m	Im3m	Pn3m		

\*The 11 Laue symmetries are separated by horizontal lines.

TABLE 1.6-5 The 65 "Biological" Space Groups

CRYSTAL SYSTEM	LAT-TICE	MINIMUM SYMMETRY OF UNIT CELL	UNIT CELL EDGES AND ANGLES*	DIFFRACTION PAT-TERN SYM-METRY*	SPACE GROUPS*
Triclinic	P	None	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$	$\bar{1}$	P1
Monoclinic	P	2-fold axis parallel to b	$a \neq b \neq c$ $\alpha = \gamma = 90^\circ$	2m	P2, P2 <sub>1</sub> , C2
	C		$\alpha = \beta = 90^\circ$		
	P		$a \neq b \neq c$		
Orthorhombic	P	3 mutually perpendicular 2-fold axes	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$	mmm	P222, P2 <sub>1</sub> 2 <sub>1</sub> 2, P222 <sub>1</sub> , P2 <sub>1</sub> 2 <sub>1</sub> 2
	C				C222, C222 <sub>1</sub>
	F				F222, F222 <sub>1</sub>
Tetragonal	P	4-fold axis parallel to c	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$	4m	P4, P4 <sub>1</sub> , P4 <sub>2</sub> , P4 <sub>3</sub>
	C				I4, I4 <sub>1</sub>
	F				F422, F422 <sub>1</sub>
Trigonal/rhombohedral	R <sup>†</sup>	3-fold axis parallel to c	$a = b = c$ $\alpha = \beta = \gamma \neq 90^\circ$	3	R3
	P <sup>†</sup>			3m	P3, P3 <sub>1</sub> , P3 <sub>2</sub>
Hexagonal	P	6-fold axis parallel to c	$a = b \neq c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$	6m	P6, P6 <sub>1</sub> , P6 <sub>2</sub> , P6 <sub>3</sub> , P6 <sub>4</sub> , P6 <sub>5</sub> , P6 <sub>6</sub>
				6mm	P622, P622 <sub>1</sub> , P622 <sub>2</sub> , P622 <sub>3</sub>
					P622 <sub>1</sub> , P622 <sub>2</sub> , P622 <sub>3</sub>
Cubic	P	3-fold axes along cube diagonals	$a = b = c$ $\alpha = \beta = \gamma = 90^\circ$	m3	P2 <sub>3</sub>
	I				I23, I23 <sub>1</sub>
	F				F23, F23 <sub>1</sub>
				m3m	P432, P432 <sub>1</sub> , P432 <sub>2</sub>
					P432
					I432, I432 <sub>1</sub> , I432 <sub>2</sub>
					F432, F432 <sub>1</sub>



## X-Ray Crystallography

*"If a picture is worth a thousand words, then a macromolecular structure is priceless to a physical biochemist."* – van Holde text

Topics:

### 1. Image Formation

Resolution / Wavelength (Amplitude, Phase) / Light Microscopy / EM / X-ray / NMR

### 2. X-Ray Crystallography

- Crystal Growth – Materials / Methods
- Crystal Lattices - Lattice Constants / Space Groups
- X-ray Sources – Sealed Tube / Rotation Anode / Synchrotron
- Theory of Diffraction – Bragg's Law / Reciprocal Space
- Data Collection – Methods / Detectors / Structure Factors
- Structure Solution – Phase Problem: MIR / MR / MAD
- Refinements and Models
- Analysis and presentation of results